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A Two-Dimensional Ray-Tracing Program

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SUMMARY

This report describes a digital-computer program, written in FORTRAN IV for the IBM 7030 (Stretch) computer, for tracing the paths of high-frequency electro-magnetic rays through two-dimensional models of the ionosphere, omitting the effects of the geomagnetic field and electron collisions from the evaluation of refractive index. The program also computes phase and group path data and absorption along the rays, and signal strength calculations are available as an option. A variety of outputs are obtainable in the form of printed tables, punched cards and ray-path plots. Each component of the program is briefly described, and FORTRAN listings and definitions of variables are provided. Examples of ionospheric models are given and the report indicates how alternative models may be written into the program.

1. INTRODUCTION

The digital-computer program described here uses a two-dimensional formulation, based on Fermat's principle, to trace the paths, in geocentric polar coordinates, of high-frequency electromagnetic rays propagated from a ground-based transmitter, through models of the ionosphere in which the electron density varies with height and range. Signal frequencies are assumed high enough for the effects of the geomagnetic field and electron collisions to be omitted from the calculation of refractive index, although a model of collision frequency is included to determine signal attenuation due to absorption. The program, written in FORTRAN IV for the IBM 7030 (Stretch) computer, is fast and capable of high accuracy under the assumptions given. Experience with the program so far indicates typical computing speeds of the order of 1 ray-hop (or 100 ray-path points) per second, at a cost of about £0.025 per ray-hop.

Development of the program was undertaken to aid investigations of various aspects of ionospheric radio-wave propagation, and it has proved to be an extremely effective tool. Examples of typical applications are the calculation of frequency deviations (Doppler shifts) due to ionospheric disturbances [1], the study of the effects of ionospheric gradients (eg, twilight zones) on the patterns of illumination on the earth by multi-hop propagation [2] and the determination of frequency variations of phase and amplitude of hf signals in studies of pulse dispersion by ionospheric propagation [3]. Although written to meet specific requirements such as those mentioned above, the program has been of use to workers in other organisations in the US and the UK.

From time to time, as fresh problems for study arose, the program underwent modification and extension, including contributions from the author's colleagues, but at each stage of development, the writer has endeavoured to ensure that, while meeting new requirements, the program could still perform previous tasks as far as possible, and to some extent anticipate others. The version presented in this report is a recent revision of the one that has seen longest use, and in order to make it available to other workers it has been written up here in a form that is a compromise between too brief a description on the one hand, and a meticulously detailed

report, which would have been too time consuming, on the other. It is felt that this compromise is adequate for interested users to run the program without difficulty.

The report commences with a statement of the equations on which the ray-tracing is based, and a note on their solution, followed by a description of the way in which ionospheric models are written into the program. After an outline of the overall program structure, there comes the bulk of the report which consists of descriptions of the component parts of the program, including FORTRAN listings of the subprograms and definitions of all variables appearing in the subprograms. As the number of comment statements in the listings was greatly increased during the recent revision of the program, the subprogram descriptions are sometimes brief, and flow diagrams, from which some subroutines would possibly benefit, have not been included. To aid the construction of data decks, the input requirements of the program are summarised, with examples applicable to the ionospheric models included with the program, and finally comments are given on the various outputs obtainable.

2. THE RAY EQUATIONS AND THEIR SOLUTION

With the assumptions that signal frequencies are high enough for the geomagnetic field and electron collisions to be omitted from the calculation of refractive index, and the electron density is variable in two dimensions only, the paths of the rays can be described by the following two equations derived from Fermat's principle of stationary phase transit time [1]:-

$$\frac{d}{ds}\left(\mu\frac{dh}{ds}\right) = \mu r\left(\frac{d\theta}{ds}\right)^2 + \frac{\partial\mu}{\partial h} \qquad \text{.....(1)}$$

and

$$\frac{d}{ds}\left(\mu r^2\frac{d\theta}{ds}\right) = \frac{\partial\mu}{\partial\theta}, \qquad \text{.....(2)}$$

s, the independent variable, is the distance along the ray,
r,θ are the geocentric polar coordinates of a point on the ray-path,
$h = r - r_e$ (r_e is the earth radius, 6370 km),
μ is the refractive index given by

$$\mu^2 = 1 - \frac{0.8061 \times 10^{-10}\,Ne}{f^2}, \qquad \text{.....(3)}$$

where f is the signal frequency (MHz), and Ne is the electron density (m^{-3}), a function of height h and range angle θ only.

By defining two other variables

$$u = \mu\frac{dh}{ds} \qquad \text{.....(4)}$$

and

$$v = \mu r^2\frac{d\theta}{ds} \qquad \text{.....(5)}$$

the ray-paths are then described by the following four differential equations:-

$$\frac{du}{ds} = \frac{v^2}{\mu r^3} + \frac{\partial\mu}{\partial h} \qquad \text{.....(6)}$$

$$\frac{dv}{ds} = \frac{\partial\mu}{\partial\theta} \qquad \text{.....(7)}$$

$$\frac{dh}{ds} = \frac{u}{\mu} \qquad \text{.....(8)}$$

and

$$\frac{d\theta}{ds} = \frac{v}{\mu r^2}. \qquad \text{.....(9)}$$

Three further quantities may be obtained from the equations:-

$$\frac{dP}{ds} = \mu \qquad \text{.....(10)}$$

$$\frac{dG}{ds} = \frac{1}{\mu} \qquad \text{.....(11)}$$

and

$$\frac{dA}{ds} = \frac{0.0461 Ne\nu}{\mu(\omega^2 + \nu^2)}, \qquad \text{.....(12)}$$

where P is the phase path,
G is the group path,
A is the absorption (db) due to electron collisions,
ω is the angular frequency, $2\pi f$ (radians/second),
and ν is the electron collision frequency (number/second).

Equations (6) to (12) are contained in a subroutine DEQS (qv) and are solved numerically by a fourth-order Runge-Kutta method (see subroutine RUKU) using pre-set values of step-length in the independent variable, s. Initial values of the variables are found by trigonometry (see subroutine LINE2B).

The values of step-length chosen depend on the circumstances of a particular case and will vary for different ionospheric models. In general, shorter step-lengths give greater accuracy but necessitate longer running times for the job. The concept of accuracy is decided by the task in hand and is not discussed here, but some comments are given, together with results of accuracy tests for some simple ionospheric models, in reference [1]. Step-lengths for a new situation are often best determined by a series of test runs in which the steps are progressively reduced until the results converge to a suitable degree.

For applications in which the ionospheric models given in this report have been used, satisfactory results have been obtained with steps ranging from 10 km in D and F-layer models, where the electron density changes relatively slowly, down to 0.1 km in a sporadic E-layer model which has a high gradient.

3. INSERTION OF IONOSPHERIC MODELS INTO THE PROGRAM

The input data for a particular ionospheric model are entered in two arrays IC and C (see blank COMMON list and summary of data input). The model derived from the input data is contained in a series of subprograms written to suit the particular case, and must include the following:-

(a) A subroutine SETC, which may be used to derive from the input data any constants of the model, eg, coefficients of equations defining model layers.

(b) A subroutine TVP, which may be used to reset any parameter of the model in a sequential manner, eg, the height of a layer, the position of a twilight zone.

(c) A function HFTH, which must define the height of a boundary in the model, eg, base, top, division between layers, given the boundary number and the relevant range angle. Usually, ionospheric boundaries are made concentric with the earth, and the function defines a series of heights, independent of range.

(d) A subroutine NRX, in which the total electron density, the refractive index, its reciprocal and derivatives, at a given height and range angle, must be defined.

Any other subroutines or functions required by the model are accessed from one or more of the above subprograms.

Examples of two types of model are given in section 6.4, and typical data inputs for them in section 7.

4. STRUCTURE OF THE RAY-TRACING PROGRAM

The following table lists the various subroutines and functions forming the program and shows how th.y are linked, each subprogram being called by the one immediately to the left and above. The subprograms fall more or less naturally into five groups, indicated by the number after each name, and these are discussed below.

```
"IŌD"(iii)
MAIN(i)

          INDATA(i)
          SETC(iv)
          TVP(iv)
          RAY2B(i)

                    LINE2B(i)
                    RUKU(i)

                              DEQS(i)

                                       NRX(iv)

                                                Subprograms for
                                                ionospheric model (iv)

                                       CŌLF(v)

                    HFTH(iv)

          PRNTPT(ii)
          PLTRAY(iii)

                    S̄ORTDA(iii)
                    PLTITL(iii)
                    PLTDAT(iii)

                              CALFRM(iii)
                              PLTFRM(iii)

          RYPRNT(ii)
          RYPNCH(ii)
          EXPHGP(ii)
          TŌTSIG(ii)
          DATAX(ii)
```

Group (i) MAIN, INDATA, RAY2B, LINE2B, RUKU and DEQS.

These form the backbone of the program and are mainly concerned with program control, data input and the tracing of the ray-paths.

Group (ii) PRNTPT, RYPRNT, RYPNCH, EXPHGP, TŌTSIG and DATAX.

These subroutines enable a variety of printed and/or punched output to be obtained, with some subsidiary computation in EXPHGP and TŌTSIG.

Group (iii) "IŌD", PLTRAY, SŌRTDA, PLTITL, PLTDAT, CALFRM and PLTFRM.

These deal with the plotting of ray-paths on the SC4060 unit. "IŌD", which is a subtype FIŌD deck, is included here as its sole purpose is the specification of the disk requirements for storing ray-data prior to plotting.

Group (iv) SETC, TVP, NRX, HFTH and subroutines containing the ionospheric model.

These subprograms control and define the particular ionospheric model used. Some or all of these may need rewriting when a different form of model is required. Two examples of models are given in this report, the first being contained in subroutines NHBP, FŌFHTH and EMH, and the second having in addition a subroutine NHES. The subprograms common to both models (in their use to date) are TVP, HFTH, FŌFHTH and EMH. Differing versions of the remaining subprograms are required by the two models (with the exception of NHES). The relevant part of the program structure is as follows:-

```
NRX

          NHBP
          FŌFHTH

                    EMH

          NHES                (Second model only)
```

Group (v) CŌLF.

This function, containing the collision frequency model, does not fit directly into any of the other groups. In its present form, as a fixed model, it could be appended to group (i), as is done in this report. However, if a different model was used, requiring read-in data, eg, through the blank CŌMMŌN array C (qv), it would be more appropriate with group (iv).

5. COMMON LISTS

A total of eight COMMON lists (seven labelled) are used in the present program for communicating data between the various subroutines. The lists are:-

5.1 COMMON (ie, blank COMMON)

5.2 COMMON/IMO/

5.3 COMMON/HOP/

5.4 COMMON/RAYLIN/

5.5 COMMON/PT/

5.6 COMMON/OP/

5.7 COMMON/PLOT/

5.8 COMMON/FRAME/

In the descriptions of the COMMON lists, the following information is given:-

(a) The subprograms using the particular list in whole or part.

(b) All items in the list in their correct order, with dimensions if necessary, and the subprograms where the items are defined.

(c) A note on each item in the list in alphabetical order.

In the notes for each subprogram, mention is made of the items used (or not used) in relevant COMMON lists, together with any local changes of names or usage, but, in general, the information given in the list descriptions is not repeated in the subroutine notes.

5.1 Blank COMMON list

Where Used

MAIN, INDATA, SETC, TVP, RAY2B, LINE2B, DEQS, NRX, HFTH, EXPHGP and in subroutines containing the ionospheric model (eg, in the examples given, NHBP, FOFHTH, EMH and NHES).

List Order	Where Defined
C(1000)	INDATA, SETC, TVP
IC(15)	INDATA, RAY2B
CF(10)	MAIN
HM	INDATA
THM	INDATA
ST(16)	INDATA

List Description

C — An array containing the data defining the particular ionospheric model used. The first NC values are input data while others are set as required in the program, eg, in SETC (qv) for values that remain constant during a run, and in TVP (qv) for values that are variable during a run. The majority of the array is at the user's disposal, but the first three elements have fixed definitions, viz,

C(1) and C(2) are the polar coordinates (r_o, θ_o) of the centre of the circles defining the boundaries of the ionospheric model. Normally, these boundaries are assumed to be concentric with the earth, and C(1) and C(2) are both input as zero. In cases where an "eccentric" model is used, C(1) is non-zero and C(2) is the angle, in radians, measured from the radius through the transmitter to the radius through the point where the ionosphere base is closest to the earth centre, the angle being positive in the direction of ray propagation (see subroutine LINE2B).

C(3) is input as the radius of the ionosphere base, ie, the lowest boundary, but after confirmatory printout, it is replaced by its square, this being the only form in which it is used (see subroutine LINE2B).

CF — An array of data that are constant for a given signal frequency f (in MHz).

CF(2) contains $0.8061 \times 10^{-10}/f^2$, used in the calculation of refractive index.

CF(3) contains $0.40305 \times 10^{-10}/f^2$, used in the calculation of the derivatives of refractive index.

CF(8) contains $4\pi^2 \times 10^{12} f^2$, the square of the angular frequency, $(\text{radians/second})^2$, used in the calculation of absorption.

CF(9) contains the factor required to obtain phase data in specified units, ie, possible values are (i) unity for phase path in km, (ii) 1/c, where c is the velocity of light in km/ms, for phase time in ms, or (iii) f/c, c in km/μs, for phase in cycles.

CF(10) contains the factor required to obtain group data in specified units, ie, possible values are (i) unity for group path in km, or (ii) 1/c, c in km/ms, for group time in ms. Note that CF(10) is independent of frequency, but is stored here because of its similarity to CF(9) which may be a function of frequency.

CF(1) and CF(4) to CF(7) are not used in the present version of the program.

HM — Maximum height, in km, that rays are allowed to reach.

IC — An array of integers for identifying and controlling the particular ionospheric model used. The first NIC values (maximum 14) are input data and the majority are at the user's disposal. The first two, however, have fixed definitions, viz,

IC(1) contains an identification number (up to three digits) for the ionospheric model in use.

IC(2) contains the number of boundaries (minimum two) in the ionospheric model, examples of boundaries being the bottom and top of the model, places where the defining equations of the model change and places where a set change in integration step-length is required (see ST below).

IC(15) is reserved, being generated within the program (in subroutine RAY2B), and contains the numerical label (from 1 to IC(2)-1) of the layer, ie, region between two boundaries, in which the current point on the ray-path lies.

ST — An array containing the set values of integration step-lengths, in km, one for each layer, ie, region between two boundaries, in the ionospheric model. The number of values is equal to IC(2)-1 with a maximum of 16, thus limiting IC(2) to a maximum of 17 (see above).

THM — Maximum range angle, in radians, which ray-paths are allowed to reach, derived from the input value of maximum range.

5.2 COMMON/IMO/list

Where Used

MAIN, INDATA, TVP, PRNTPT, PLTRAY, PLTITL, RYPRNT, RYPNCH, EXPHGP, TOTSIG and DATAX.

List Order	Where defined
JOBNO	INDATA
IID	INDATA
LT	MAIN
F	MAIN
NA	INDATA, MAIN
NH	INDATA
DA	INDATA, MAIN
ALPH1	INDATA, MAIN
KP	INDATA
KG	INDATA
KX	INDATA
JP	INDATA
JX	INDATA
HEF	INDATA
NTF	INDATA
NTL	INDATA
NDT	INDATA
NF	INDATA
KA	INDATA
MPT	INDATA
MPLT	INDATA
MPRNT	INDATA
MPUN	INDATA
MXPG	INDATA
MSIG	INDATA
MXDAT	INDATA
FREQ(24)	INDATA

List Description

ALPH1 Takeoff angle, in degrees, of the first ray in a set.

DA Increment in ray takeoff angles, in degrees.

F Current value of signal frequency, in MHz.

FREQ Array containing the input signal frequencies, in MHz.

HEF Height, in km, at which a mode split takes place. The present program can only accomodate one such split, eg, normally between E and F layers.

IID Identification number for the ionospheric model in use, obtained from IC(1) (see blank COMMON list).

JŌBNŌ An integer of up to four digits identifying a particular run.

JP Control for the ordering of the punching of RAYSET cards in subroutine RYPNCH. Possible values are:

1 - output is cycled on hops, ie, hop 1, ray 1; hop 1, ray 2; ....; hop 1, ray N; hop 2, ray 1; ....; hop 2, ray N; etc.

2 - output is cycled on rays, ie, ray 1, hop 1; ray 1, hop 2; ....; ray 1, hop M; ray 2, hop 1; ....; ray 2, hop M; etc.

JX Control for the ordering of the data output (print and punch) in subroutine DATAX. Possible values are as for JP above.

KA Control for the reading of ray takeoff angle data (NA, ALPH1, DA). Possible values are:

1 - angle data is constant for all time-steps and frequencies, and only one set is read (in subroutine INDATA).

2 - angle data may vary with time-step but is constant for all frequencies within each time-step, so a set is read (in MAIN program) on each pass through the time-step loop.

3 - angle data may be different for each time-step/frequency combination, so a set is read (in MAIN program) on each pass through the frequency loop.

KG Control for determining the units of the computed group data. Possible values are:

1 - group path in km.

2 - group time in ms.

KP Control for determining the units of the computed phase data. Possible values are:

1 - phase path in km.

2 - phase time in ms.

3 - phase in cycles.

KX Control for determining the units of excess phase and group data. Possible values are from 1 to 6, viz,

1,2 - excess phase path in km.

3,4 - excess phase time in μs.

5,6 - excess phase in cycles.

1,3,5 - excess group path in km.

2,4,6 - excess group time in μs.

LT — Current time-step number.

MPLT — Control for the plotting of the computed ray-paths (subroutine PLTRAY). Possible values are:

1 - enabled, or 2 - inhibited.

MPRNT — Control for the printout of a data summary for each ray-hop (subroutine RYPRNT). Possible values are:

1 - enabled, or 2 - inhibited.

MPT — Control for the printout of data for each point in a ray-hop (subroutine PRNTPT). Possible values are:

1 - enabled, or 2 - inhibited.

MPUN — Control for the punching of RAYSET cards (subroutine RYPNCH). Possible values are:

1 - enabled, or 2 - inhibited.

MSIG — Control for the calculation and printout of total signal strength data (subroutine TŌTSIG). Possible values are:

1 - enabled, or 2 - inhibited.

MXDAT — Control for the printing and punching of excess phase and group data and total signal strength (subroutine DATAX). Possible values are:

1 - enabled (if subroutines EXPHGP and TŌTSIG have both been enabled), or 2 - inhibited.

MXPG — Control for the calculation and printout of excess phase and group data (subroutine EXPHGP). Possible values are:

1 - enabled, or 2 - inhibited.

NA — Number of takeoff angles (ie, rays) in a set, maximum 100.

NDT — Increment in time-step number.

NF — Number of input signal frequencies, maximum 24.

NH — Maximum number of hops (up to 5) in any ray.

NTF — First required time-step number.

NTL — Last required time-step number.

5.3 COMMON/HOP/list

Where Used

MAIN, RAY2B, LINE2B, PRNTPT and PLTRAY.

List Order	Where Defined
ALPHR	MAIN, RAY2B, LINE2B
NDR	RAY2B, LINE2B
LA	MAIN
LH	MAIN
I	RAY2B
H(1000)	RAY2B
THETA(1000)	RAY2B
RHO(1000)	RAY2B
S	RAY2B
PH(1000)	RAY2B
GP(1000)	RAY2B
AB(1000)	RAY2B
ALPHA	MAIN
ALPHD	MAIN
IPK	RAY2B
HIPK	RAY2B
RHOIPK	RAY2B
HND	RAY2B
RHOND	RAY2B
PHND	RAY2B
GPND	RAY2B
ABND	RAY2B

List Description

AB — Array containing values of absorption, in db, to each point in a ray-hop.

ABND — The absorption, in db, to the end of a ray-hop.

ALPHA — The initial takeoff angle of a ray, in degrees.

ALPHD — The elevation angle of a ray at the end of a hop, in degrees.

ALPHR — The elevation angle of a ray at the beginning or end of a hop, in radians.

GP — Array containing values of group path, in km, or group time, in ms, to each point in a ray-hop.

GPND — The value of group path, in km, or group time, in ms to the end of ray-hop.

H — Array containing the heights, in km, of each point in a ray-hop.

HIPK — The height, in km, of the highest point reached in a ray-hop, normally the apogee but it may be the end height if the ray escapes or if the last hop is incomplete.

HND — The height, in km, at the end of a ray-hop, normally zero unless the hop ends in a perigee or is incomplete, or the ray escapes.

I — Total number of points computed in a ray-hop.

IPK — Subscript of the highest point reached in a ray-hop, normally of the apogee but may be equal to I if the last hop is incomplete or the ray escapes.

LA — Current takeoff angle (ie, ray) number.

LH — Current hop number.

NDR — Control indicating how a ray-hop ended. Possible values are:-

1 - ray returned to earth surface,

2 - ray reached a perigee below the ionosphere,

3 - ray reached a perigee in the ionosphere,

4 - ray terminated at maximum height,

5 - ray terminated at maximum range,

6 - ray terminated due to maximum number of points (1000) being approached in current hop.

PH — Array containing values of phase path, in km, phase time, in ms, or phase, in cycles, to each point in a ray-hop.

PHND — The value of phase path, in km, phase time, in ms, or phase, in cycles, to the end of a ray-hop.

RHŌ — Array containing values of great circle range, in km, to each point in a ray-hop.

RHŌIPK — The great circle range, in km, of the highest point reached in a ray-hop, normally of the apogee but it may be of the end point if the ray escapes or if the hop is incomplete.

RHŌND — The great circle range, in km, to the end of a ray-hop.

S — Ray-path length, in km, to the end of a ray-hop.

THETA — Army containing values of range angle, in radians, to each point in a ray-hop.

5.4 COMMON/RAYLIN/list

Where Used

RAY2B and LINE2B.

List Order	Where defined
N	RAY2B
H1	RAY2B
THETA1	RAY2B
HJ	RAY2B, LINE2B
DTH	RAY2B, LINE2B
DS	LINE2B
U	RAY2B, LINE2B
V	RAY2B, LINE2B

List Description

DS — Length, in km, of linear section of ray-path.

DTH — Increment in range angle, in radians, over linear section of ray-path.

HJ — Height, in km, at end of linear section of ray-path.

H1 — Height, in km, at start of linear section of ray-path.

N — Control to select required linear section of ray-path. Possible values are:-

1 - ascent to base of ionospheric model,

2 - descent from base of ionospheric model to earth surface or perigee,

3 - ascent from top of ionospheric model to maximum height,

4 - ascent, below ionosphere, to maximum range,

5 - descent from ionosphere base to maximum range,

6 - ascent from top of ionosphere to maximum range.

THETA1 — Range angle, in radians, to start of linear section of ray-path.

U — Value of $\mu dh/ds$ at start or end of linear section of ray-path (μ = refractive index, h = height, s = distance along ray).

V — Value of $\mu r^2 d\theta/ds$ at start or end of linear section of ray-path (r = radius, θ = range angle).

5.5 COMMON/PT/list

Where Used

DEQS, NRX and in the subroutines containing the ionospheric model (eg, in the examples given, NHBP, FOFHTH, EMH and NHES).

List Order	Where Defined
H	DEQS
THETA	DEQS
EN	NRX
EMU	NRX
RMU	NRX
DMUDH	NRX
DMUDTH	NRX
*	
ENH	NHBP
DNDH	NHBP
FHTH	FOFHTH
DFDH	FOFHTH
DFDTH	FOFHTH
EM	EMH
DMDH	EMH
ENS	NHES
DNSDH	NHES

* Note: The second part of the list has been written to accomodate ionospheric models of the forms given in the examples. Other forms of model, requiring different versions of subroutine NRX and the subroutines containing the model, may well necessitate alterations or extensions to this part of the list. Accordingly, the list description below is divided into two parts, the first being fixed (for the present version of subroutine DEQS, anyway), and the second being that applicable to the current ionospheric model subroutines.

List Description, Part 1

DMUDH — Derivative of refractive index with respect to height.

DMUDTH — Derivative of refractive index with respect to range angle.

EMU — Refractive index at the current values of height and range angle.

EN — Electron density, electrons/m^3, at the current values of height and range angle.

H — Current value of height, in km.

RMU — Reciprocal of refractive index.

THETA — Current value of range angle, in radians.

List Description, Part 2

DFDH	Derivative of the twilight transition function, $F(h,\theta)$, with respect to height.
DFDTH	Derivative of the twilight transition function, $F(h,\theta)$, with respect to range angle.
DMDH	Derivative of the function, $m(h)$, defining the ratio of night to day electron densities.
DNDH	Derivative of the basic (day-time) electron density profile, $N(h)$.
DNSDH	Derivative of the sporadic E-layer electron density function, $N_{ES}(h)$.
EM	Ratio of night to day electron densities, $m(h)$, at the current value of height.
ENH	Electron density, electrons/m^3, in the basic (day-time) ionospheric profile, $N(h)$, at the current value of height.
ENS	Electron density, electrons/m^3, in the sporadic E-layer model, $N_{ES}(h)$, at the current value of height.
FHTH	Value of the twilight transition function, $F(h,\theta)$, at the current values of height and range angle.

5.6 COMMON/OP/list

Where Used

MAIN, RYPRNT, RYPNCH, EXPHGP, TOTSIG and DATAX.

List Order	Where Defined
KEY(5,100)	MAIN
NDRL(5,100)	MAIN
RHOL(5,100)	MAIN
ALPHAS(100)	MAIN
PHL(5,100)	MAIN
GPL(5,100)	MAIN
ABL(5,100)	MAIN
HPK(5,100)	MAIN
SL(5,100)	MAIN
HL(5,100)	MAIN
ALPHDL(5,100)	MAIN
IL(5,100)	MAIN
RHOK(5,100)	MAIN
ALPHRS(100)	MAIN
ALPHRL(5,100)	MAIN
XPH(5,100)	EXPHGP
XGP(5,100)	EXPHGP
SIGL(5,100)	TOTSIG

List Description

ABL — Array containing values of absorption, in db, to the end of each ray-hop.

ALPHAS — Array containing the initial takeoff angles, in degrees, of each ray in a set.

ALPHDL — Array containing the elevation angles, in degrees, at the end of each ray-hop.

ALPHRL — Array containing the elevation angles, in radians, at the end of each ray-hop.

ALPHRS — Array containing the initial takeoff angles, in radians, of each ray in a set.

GPL — Array containing values of group path, in km, or group time, in ms, to the end of each ray-hop.

HL — Array containing the values of height, in km, at the end of each ray-hop.

HPK — Array containing the values of the greatest height, in km, achieved in each ray-hop.

IL — Array containing the numbers of points computed in each ray-hop.

KEY	Array of controls indicating that a given ray-hop was (value 1) or was not (value 0) computed.
NDRL	Array of controls indicating how each ray-hop ended. Possible values and their meanings are given under NDR in the COMMON/HOP/ list.
PHL	Array containing values of phase path, in km, phase time, in ms, or phase, in cycles, to the end of each ray-hop.
RHOK	Array containing the values of great circle range, in km, corresponding to the highest point in each ray-hop.
RHOL	Array containing the values of great circle range, in km, to the end of each ray-hop.
SIGL	Array containing the values of signal strength, in db, at the end of each ray-hop.
SL	Array containing the values of ray path length, in km, to the end of each ray-hop.
XGP	Array containing the values of excess group path, in km, or excess group time, in μs, at the end of each ray-hop.
XPH	Array containing the values of excess phase path, in km, excess phase time, in μs, or excess phase, in cycles, at the end of each ray-hop.

5.7 COMMON/PLOT/list

Where Used

INDATA, PLTRAY, SORTDA, PLTITL, PLTDAT, CALFRM and PLTFRM.

List Order	Where Defined
NTYP	INDATA
NTITLE	INDATA
TITLE(400)	INDATA
MAXH	INDATA
INCDH	INDATA
IPTS	PLTRAY
HTS(5000)	PLTRAY, PLTDAT
THS(5000)	PLTRAY, PLTDAT
NUMSEC(15)	PLTRAY, SORTDA
NPTSEC(15,100)	PLTRAY, SORTDA
NFR	PLTRAY, SORTDA

List Description

HTS — Array containing the values of height, in km, of each point in a ray-path which may extend over several hops.

INCDH — Increment in height scale annotations, in km (integer).

IPTS — Number of points in a ray-path which may extend over several hops.

MAXH — Maximum height, in km (integer and a multiple of INCDH above), of a plotting frame.

NFR — Number of frames required for plotting a set of ray-paths.

NPTSEC — Array containing the numbers of points in each section of a ray-path in each frame.

NTITLE — Number of A8 words in the array TITLE (below). Maximum 400, ie, up to 40 cards.

NTYP — Control indicating the plotting medium required. Possible values are:

1 - hard copy, 2 - microfilm, 3 - both.

NUMSEC — Array containing the numbers of ray-path sections to be plotted in each frame.

THS — Array containing the values of range angle, in radians, to each point in a ray-path which may extend over several hops.

TITLE — Array of A8 words containing a title or description to be printed on the page preceding each set of ray plots.

5.8 COMMON/FRAME/list

Where Used

PLTDAT, CALFRM and PLTFRM.

List Order	Where Defined
LRS	PLTDAT
LRF	PLTDAT
RHX	CALFRM
NHS	CALFRM
LHS(9)	CALFRM
YHL(9)	CALFRM
XLR(9), YLR(9)	CALFRM
XCL(9), YCL(9)	CALFRM
XHC(9), YHC(9)	CALFRM
XCR(9), YCR(9)	CALFRM
XRL(9), YRL(9)	CALFRM
XHR(9), YHR(9)	CALFRM
XRO(101), YRO(101)	CALFRM
XRM(101), YRM(101)	CALFRM
XOA(10), YOA(10)	CALFRM
XOB(10), YOB(10)	CALFRM
XMA(10), YMA(10)	CALFRM
XMB(10), YMB(10)	CALFRM
XRX, YRX	CALFRM

Note: With the exception of YHL, "Y" coordinates are paired below with the corresponding "X" coordinates.

List Description

LHS — Array containing the height values, in km, for annotating the height scale.

LRF — Range value, in km, for annotating the right hand end of the range scale.

LRS — Range value, in km, for annotating the start of the range scale.

NHS — Number of height scale marks (excluding zero and maximum height marks), maximum 9.

RHX — Height scale factor, plotter units/km.

XCL, YCL — Arrays containing the plotter coordinates of the left ends of the frame-centre height-scale marks.

XCR, YCR — Arrays containing the plotter coordinates of the right ends of the frame-centre height-scale marks.

XHC, YHC — Arrays containing the plotter coordinates of the centres of the frame-centre height-scale marks.

XHR, YHR Arrays containing the plotter coordinates of the right ends of the frame-right height-scale marks.

XLR, YLR Arrays containing the plotter coordinates of the right ends of the frame-left height-scale marks.

XMA, YMA Arrays containing the plotter coordinates of the scale ends of the upper range-scale marks.

XMB, YMB Arrays containing the plotter coordinates of the upper ends of the upper range-scale marks.

$X\bar{O}A$, $Y\bar{O}A$ Arrays containing the plotter coordinates of the scale ends of the surface range-scale marks.

$X\bar{O}B$, $Y\bar{O}B$ Arrays containing the plotter coordinates of the lower ends of the surface range-scale marks.

XRL, YRL Arrays containing the plotter coordinates of the left ends of the frame-right height-scale marks.

XRM, YRM Arrays containing the plotter coordinates of the points forming the range-scale at the frame upper edge (ie, at maximum height).

$XR\bar{O}$, $YR\bar{O}$ Arrays containing the plotter coordinates of the points forming the surface range scale.

XRX, YRX Plotter coordinates of the start of the annotation of the right hand mark on the surface range-scale.

YHL Array containing the "y" plotter coordinates of the annotations and scale marks on the height-scale.

6. SUBPROGRAM DESCRIPTIONS AND LISTINGS

The subprograms are divided into sets corresponding with the groupings mentioned in section 4, viz,

6.1 Group (i) and group (v) subprograms.

6.2 Group (ii) subroutines.

6.3 Group (iii) subprograms.

6.4 Group (iv) subprograms.

For each subprogram, the following are provided:-

(a) A description.

(b) A dictionary of definitions of arguments (if any) and local variables, and also a reference to any COMMON variables used (for definitions, see section 5), with notes on any local changes of name or usage.

(c) A FORTRAN listing.

6.1 Group (i) and group (v) subprograms

6.1.1 MAIN program.

6.1.2 Subroutine INDATA.

6.1.3 Subroutine RAY2B.

6.1.4 Subroutine LINE2B.

6.1.5 Subroutine RUKU

6.1.6 Subroutine DEQS.

6.1.7 Function COLF.

6.1.1 MAIN program

After calling subroutine INDATA (qv) to read the data input for a run, and subroutine SETC (qv) to set up the ionospheric model to be used, the principle operations that take place in the MAIN program are the organisation of the ray computations and the selection of a variety of outputs.

The ray computations take place in the inner of four nested DŌ-loops, each of which supplies data necessary to the computations.

The outer or "time-step" loop allows, if required, the variation of a parameter or parameters of the particular ionospheric model used, by a call to the subroutine TVP (qv) on each pass through the loop. Examples of such parameters are the height of a layer, the position of a twilight transition, the time during the decay of an ionisation impulse, etc.

The second or frequency loop, selects in turn each of the signal frequencies input to the program (present maximum is 24 frequencies), and a number of frequency dependent coefficients are defined.

The third or angle loop, sets the initial takeoff angle of a ray-path. At present, up to 100 takeoff angles, ie, rays, are permitted in any set.

On each pass through the inner or hop loop, one hop of a ray-path is computed by calling subroutine RAY2B (qv). Up to a maximum (at present) of 5 hops may be requested, although the ray may be terminated earlier if the maximum height or range is reached, or if a hop requires more than the maximum permitted number of points (1000).

At the end of each hop, significant data about it, eg, apogee and end point data, are stored for later use. A table of the computed data for each point in the hop may be obtained, if required, by a call to subroutine PRNTPT (qv). Also, the hop data (height and range angle for each point) may be stored for plotting, by a call to subroutine PLTRAY (qv).

After the completion of the last ray path in a set (ie, time-step/frequency combination), further outputs may be obtained by calls to some or all of the subroutines RYPRNT, RYPNCH, EXPHGP, TŌTSIG and DATAX, the type of output in each case being given in the notes about the particular subroutine.

STORAGE LIST

Local Variables

DTR — Factor to convert degrees to radians, ie, $\pi/180$. Set in DATA statement.

FCSQŌN — Constant appearing in the refractive index equation, ie, 0.8061×10^{-10} (when electron density is in electrons/m^3 and signal frequency in MHz). Set in DATA statement.

FSQ — Square of signal frequency (in MHz).

LF — DŌ-loop index for signal frequencies.

PI2MSQ — Factor to convert frequency (in MHz) squared to angular frequency (radians/s) squared, ie, $4\pi^2 \times 10^{12}$. Set in DATA statement.

RTD — Factor to convert radians to degrees, ie, $180/\pi$. Set in DATA statement.

UPG — Array of constants for the conversion factors used to obtain phase and group data in required units from the values in km. Set in DATA statement. The constants are:-

- UPG(1) — Unity, ie, phase and/or group paths are in km, as originally computed.
- UPG(2) — 1/c, where c is the velocity of light in km/ms, ie, phase and/or group times are obtained, in ms,
- UPG(3) — 1/c, where c is the velocity of light in km/μs; when multiplied by the signal frequency (in MHz), this enables phase to be obtained in cycles.

Blank CŌMMŌN

The only item in the list (qv) referenced here is the array CF, elements 2, 3, 8, 9 and 10 of which are defined in this program.

CŌMMŌN/IMŌ/

The full list (qv) is used with the exception of HEF, IID, JŌBNŌ, JP, JX and KX.

CŌMMŌN/HŌP/

The full list (qv) is used with the exception of AB, GP, H, IPK, PH, RHŌ and THETA.

CŌMMŌN/ŌP/

All items in the list (qv) up to and including ALPHRL are defined in this program.

```
C     RAY PATHS IN A TWO-DIMENSIONAL IONOSPHERE
C     -----------------------------------------
C     ORIGINAL VERSION NOVEMBER 67 A.R.C.
C     MODIFIED FOR PLOTTING FEBRUARY 68 R.M.J.
C     MODIFIED TO EXTEND OUTPUTS OCTOBER 68 K.B.B.
C     REVISION 27/10/69 A.R.C.
C
      COMMON C(1000),IC(15),CF(10)
      COMMON /IMO/ JOBNO,IID,LT,F,NA,NH,DA,ALPH1,KP,KG,KX,JP,JX,HEF,NTF,
     1             NTL,NDT,NF,KA,MPT,MPLT,MPRNT,MPUN,MXPG,MSIG,MXDAT,
     2             FREQ(24)
      COMMON /HOP/ ALPHR,NDR,LA,LH,I,H(1000),THETA(1000),RHO(1000),S,
     1             PH(1000),GP(1000),AB(1000),ALPHA,ALPHD,IPK,HIPK,
     2             RHOIPK,HND,RHOND,PHND,GPND,ABND
      COMMON /OP/ KEY(5,100),NDRL(5,100),RHOL(5,100),ALPHAS(100),
     1            PHL(5,100),GPL(5,100),ABL(5,100),HPK(5,100),SL(5,100),
     2            HL(5,100),ALPHDL(5,100),IL(5,100),RHOK(5,100),
     3            ALPHRS(100),ALPHPL(5,100)
C
C     FACTORS TO OBTAIN PHASE AND GROUP DATA IN REQUIRED UNITS
      DIMENSION UPG(3)
      DATA UPG(1)(1.),UPG(2)(.3335640484668 6E-2),UPG(3)(3.3356404846686)
C     CONVERSION FACTORS, DEGREES-TO-RADIANS AND RADIANS-TO-DEGREES
      DATA DTR(1.7453292519943E-2),RTD(57.295779513082)
C     CONSTANT IN REFRACTIVE INDEX EQUATION (FCSCON), FACTOR TO CONVERT
C     F(MHZ) SQUARED TO OMEGA(RADIANS/SEC) SQUARED (PI2MSQ)
      DATA FCSCON(.8061E-10),PI2MSQ(39.478417604357E+12)
C     LIBRARY ROUTINE TO DUMP CORE CONTENTS IN EVENT OF ABEOJ
      CALL EDUMP
C     PREPARE SC4060 PLOTTING ROUTINES (LIBRARY)
      CALL SCLIBR
C     READ INPUT DATA FOR THIS RUN
      CALL INDATA
C     SET REMAINING CONSTANTS C FOR IONOSPHERIC MODEL
      CALL SETC
C     SET CF VALUES TO COMPUTE PHASE AND GROUP DATA IN (1)KM OR (2)MSEC
      CF(9)=UPG(KP)
      CF(10)=UPG(KG)
C     EXECUTE THE FOLLOWING LOOP FOR EACH REQUIRED TIME-STEP
      DO 20 LT=NTF,NTL,NDT
C     SET TIME DEPENDENT PARAMETERS FOR THIS TIME-STEP
      CALL TVP
C     READ ANGLE DATA IF CONSTANT FOR ALL FREQS AT THIS TIME (KA=2)
      IF(KA.EQ.2)READ 4001,NA,ALPH1,DA
C     EXECUTE THE FOLLOWING LOOP FOR EACH INPUT SIGNAL FREQUENCY
      DO 19 LF=1,NF
      F=FREQ(LF)
C     REVISE CF(9) IF PHASE IS REQUIRED IN CYCLES (KP=3)
      IF(KP.EQ.3)CF(9)=CF(9)*F
C     SET SQUARE OF ANGULAR FREQUENCY IN CF(8) FOR COMPUTING ABSORPTION
      FSQ=F*F
      CF(8)=FSQ*PI2MSQ
C     CF(2),CF(3) ARE USED IN COMPUTING REFRACTIVE INDEX AND DERIVATIVES
      CF(2)=FCSCON/FSQ
      CF(3)=CF(2)*.5
```

```
C     READ ANGLE DATA IF DIFFERENT FOR EACH FREQ AT EACH TIME (KA=3)
      IF(KA.EQ.3)READ 4001,NA,ALPH1,DA
C     INITIALISE RAY-HOP INDICATORS (KEY)
      DO 2 LA=1,NA
      DO 1 LH=1,NH
      KEY(LH,LA)=0
    1 CONTINUE
    2 CONTINUE
C     INITIALISE TAKEOFF ANGLE AND EXECUTE THE FOLLOWING LOOP FOR EACH
C     OF THE REQUIRED TAKEOFF ANGLES (RAYS)
      ALPHA=ALPH1-DA
      DO 8 LA=1,NA
C     SET TAKEOFF ANGLE, CONVERT TO RADIANS AND STORE BOTH FORMS FOR
C     LATER USE
      ALPHA=ALPHA+DA
      ALPHR=ALPHA*DTR
      ALPHRS(LA)=ALPHR
      ALPHAS(LA)=ALPHA
C     COMPUTE EACH RAY PATH FOR UP TO NH HOPS
      DO 7 LH=1,NH
      CALL RAY2B
C     RESET APPROPRIATE KEY TO INDICATE THAT RAY-HOP HAS BEEN COMPUTED
      KEY(LH,LA)=1
C     CONVERT END OF RAY ANGLE TO DEGREES
      ALPHD=RTD*ALPHR
C     STORE SIGNIFICANT HOP DATA FOR LATER USE AND OUTPUT
      ALPHDL(LH,LA)=ALPHD
      ALPHRL(LH,LA)=ALPHR
      HPK(LH,LA)=HIPK
      RHOK(LH,LA)=RHOIPK
      HL(LH,LA)=HND
      RHOL(LH,LA)=RHOND
      SL(LH,LA)=S
      PHL(LH,LA)=PHND
      GPL(LH,LA)=GPND
      ABL(LH,LA)=ABND
      NDRL(LH,LA)=NDR
      IL(LH,LA)=I
C     PRINT THE DATA FOR EVERY POINT IN THE HOP IF REQUIRED
      GO TO (3,4),MPT
    3 CALL PRNTPT
C     CALL PLOTTING ROUTINE IF PLOTTED OUTPUT REQUIRED
    4 GO TO (5,6),MPLT
    5 CALL PLTRAY
C     IF RAY REACHED MAX HEIGHT (NDR=4), MAX RANGE (NDR=5) OR IF POINT
C     COUNT REACHED MAX (NDR=6), START ANOTHER RAY. OTHERWISE COMPUTE
C     ANOTHER HOP IF REQUIRED
    6 IF(NDR-4)7,8,8
C     END OF HOP LOOP
    7 CONTINUE
C     END OF ANGLE (RAY) LOOP
    8 CONTINUE
```

```
C     PRINT RAYSETS IF REQUIRED
      GO TO (9,10),MPRNT
    9 CALL RYPRNT
C     PUNCH RAYSETS IF REQUIRED
   10 GO TO (11,12),MPUN
   11 CALL RYPNCH
C     CALCULATE AND PRINT EXCESS PHASE AND GROUP DATA IF REQUIRED
   12 GO TO (13,14),MXPG
   13 CALL EXPHGP
C     CALCULATE AND PRINT TOTAL SIGNAL STRENGTH DATA IF REQUIRED
   14 GO TO (15,19),MSIG
   15 CALL TOTSIG
C     LIST AND PUNCH CROSS-DATA IF REQUIRED. (ONLY VALID IF SUBROUTINES
C     EXPHGP AND TOTSIG HAVE BOTH BEEN CALLED PREVIOUSLY)
      GO TO (16,19),MXDAT
   16 GO TO (17,18),MXPG
   17 CALL DATAX
      GO TO 19
C     INVALID REQUEST FOR CROSS-DATA
   18 PRINT 4101
C     END OF FREQUENCY LOOP
   19 CONTINUE
C     END OF TIME-STEP LOOP
   20 CONTINUE
C     JOB COMPLETED. FINISH WITH SC4060
      CALL FINISH
      RETURN
 4001 FORMAT(I8,2F8.3)
 4101 FORMAT(86H1*****REQUEST FOR CROSS-DATA IS INVALID. EXCESS PHASE AN
     1D GROUP DATA NOT COMPUTED*****)
      END
```

6.1.2 Subroutine INDATA

This subroutine is used to read in all the data required for a run, with the exception that when KA (see COMMON/IMO/list) has the value 2 or 3, reading of ray takeoff angle data takes place in the MAIN program (qv).

Some of the input data, eg, ionospheric model parameters, are printed out to form a title for the output data.

A summary of the required input data is given in section 7 together with examples of data decks.

STORAGE LIST

Local Variables

FMI	Array of A8 words (maximum 100, ie, up to 10 data cards) defining the FORMAT for printing the input ionospheric data.
J	Loop index for reading and printing data input to arrays.
NC	Number of values to be read into array C (see blank COMMON list).
NFMI	Number of A8 words to be read into array FMI (above).
NIC	Number of values to be read into array IC (see blank COMMON list).
NS	Number of values to be read into array ST (see blank COMMON list).
RHOM	Maximum range, in km, to which rays may be traced.

Blank COMMON

All items in the list (qv), with the exception of array CF, are referenced here.

COMMON/IMO/

All items in the list (qv), with the exception of LT and F, are referenced here.

COMMON/PLOT/

The first five items in the list (qv) are input here.

```
      SUBROUTINE INDATA
C     DATA INPUT FOR RUN
C     VERSION A (REVISION 28/10/69 A.R.C. OF SUB INPUT 14/2/69 K.B.B.)
C
      COMMON C(1000),IC(15),CF(10),HM,THM,ST(16)
      COMMON /IMO/ JOBNO,IID,LT,F,NA,NH,DA,ALPH1,KP,KG,KX,JP,JX,HEF,NTF,
     1             NTL,NDT,NF,KA,MPT,MPLT,MPRNT,MPUN,MXPG,MSIG,MXDAT,
     2             FREQ(24)
      COMMON /PLOT/ NTYP,NTITLE,TITLE(400),MAXH,INCDH
      DIMENSION FMI(100)
C
C     READ CONTROLS FOR THIS RUN
      READ 4001,JOBNO,NTF,NTL,NDT,NF,NH,KA,KP,KG,KX,JP,JX
C     READ OUTPUT OPTIONS
      READ 4001,MPT,MPLT,MPRNT,MPUN,MXPG,MSIG,MXDAT
C     READ AND PRINT IONOSPHERE MODEL DATA
      READ 4001,NC,NFMI,NIC,(IC(J),J=1,NIC)
      READ 4002,(C(J),J=1,NC)
      READ 4003,(FMI(J),J=1,NFMI)
```

```
      PRINT FMI,(IC(J),J=1,NIC),(C(J),J=1,NC)
C     SET IONOSPHERE MODEL NUMBER IID (IC(1)). SET NUMBER OF INTEGRATION
C     STEP LENGTHS NS (IC(2)-1). REPLACE C(3) BY ITS SQUARE (FOR LINE2B)
      IID=IC(1)
      NS=IC(2)-1
      C(3)=C(3)*C(3)
C     READ AND PRINT INTEGRATION STEP LENGTHS, ONE FOR EACH LAYER (I.E.
C     REGION BETWEEN BOUNDARIES) OF THE IONOSPHERIC MODEL
      READ 4004,(ST(J),J=1,NS)
      PRINT 4101,(J,ST(J),J=1,NS)
C     READ AND PRINT HEIGHT AND RANGE LIMITS. (ALSO READ HEIGHT OF SPLIT
C     BETWEEN MODES, E.G. E AND F, ES AND F, FOR TOTSIG)
      READ 4002,HM,RHOM,HEF
      PRINT 4102,HM,RHOM,NH
C     SET MAXIMUM RANGE ANGLE
      THM=RHOM/6370.
C     READ THE FREQUENCIES REQUIRED FOR THIS RUN
      READ 4004,(FREQ(J),J=1,NF)
C     IF PLOTTED OUTPUT IS REQUIRED, READ PLOTTING CONTROLS AND TITLE
      GO TO (1,2),MPLT
    1 READ 4001,NTYP,NTITLE,MAXH,INCDH
      READ 4003,(TITLE(J),J=1,NTITLE)
C     READ ANGLE DATA IF CONSTANT FOR ALL TIME-STEPS AND FREQS (KA=1)
    2 IF(KA.EQ.1)READ 4005,NA,ALPH1,DA
C     IF KA=2 OR 3 ANGLE DATA IS READ IN MAIN PROGRAM
      RETURN
C     INPUT DATA FORMATS
 4001 FORMAT(16I5)
 4002 FORMAT(8E10.5)
 4003 FORMAT(10A8)
 4004 FORMAT(8F10.6)
 4005 FORMAT(I8,2F8.3)
C     PRINTOUT FORMATS
 4101 FORMAT(1H ,9X20HSTEP LENGTH IN LAYER,I3,3H IS,F8.3,3H KM)
 4102 FORMAT(1H0,9X31HRAY LIMITS      MAXIMUM HEIGHT =,F7.1,23H KM      MA
     1XIMUM RANGE =,F8.1,6H KM OR,I2,5H HOPS)
      END
```

6.1.3 Subroutine RAY2B

This subroutine traces out one hop of the ray-path. Most frequently, this will be from ground-to-ground via reflection in the ionospheric model, but may, in some cases, be from ground-to-perigee, perigee-to-perigee or perigee-to-ground, the latter two instances only being possible for hops other than the first of a ray since the program assumes a ground-based ray-source. Perigees are divided into two types, those occurring below the ionosphere and those in the ionosphere. The paths described above may be termed complete hops.

Hops may be terminated before completion if (i) the ray reaches a maximum specified height (in such cases the ray would not normally reflect anyway, as the maximum height specified is usually above the maximum height of reflection), (ii) the ray reaches a maximum specified range, or (iii) the number of points computed nears the maximum storage allowed (1000 points). When any of these three cases arise, a control NDR (qv) indicates to the calling program (MAIN) that no more hops can be computed for this particular ray.

End points of linear sections of the ray-path, ie, those sections in regions below or above the ionospheric model where the refractive index is assumed constant, are obtained by a call to the subroutine LINE2B.

In the region covered by the ionospheric model, successive points along the ray-path are computed by the fourth-order Runge-Kutta method contained in subroutine RUKU. After each point is obtained, checks are made to determine if some significant "feature" of the ray-path has been passed, a "feature" being one of the following: (i) apogee, (ii) perigee, (iii) maximum height, (iv) maximum range, or (v) boundary crossing. If one of these has been passed, it is located to within a specified tolerance by adjusting the integration step-length by an iterative inverse linear interpolation process.

After an apogee, ie, ionospheric reflection point, is located, the ray is continued on its downward path. When a perigee, maximum height or maximum range is located, the hop or ray is ended as mentioned above. After a boundary crossing, ie, a level where the equations defining the ionospheric model change and/or a change in integration step-length is required, the ray is continued, either with the new step-length or linearly, as appropriate.

STORAGE LIST

Local Variables

BH	Height, in km, of an ionospheric boundary.
DAB	Increment in absorption for an increment in ray-path length. (Note multiplication by 0.0461 to convert to db.) Equivalent to DQ(7) (qv).
DGP	Increment in group path, in km, for an increment in ray-path length. Equivalent to DQ(6) (qv).
DH	Increment in height, in km, for an increment in ray-path length. Equivalent to DQ(3) (qv).
DPH	Increment in phase path, in km, for an increment in ray-path length. Equivalent to DQ(5) (qv).
DQ	Array containing the increments in the ray variables Q (qv) for an increment in ray-path length, the independent variable. The array elements are referenced by equivalent names: DU, DV, DH, DTHET, DPH, DGP and DAB (qv).
DTHET	Increment in range-angle, in radians, for an increment in ray-path length. Equivalent to DQ(4) (qv).
DU	Increment in the variable u (see UI) for an increment in ray-path length. Equivalent to DQ(1) (qv).
DV	Increment in the variable v (see VI) for an increment in ray-path length. Equivalent to DQ(2) (qv).
DYA	Difference between the value of a variable defining a feature on the ray-path, eg, apogee, boundary crossing, and the last value computed before the feature. Used in the procedure for locating a feature.
DYC	Difference between the value of a variable defining a ray-path feature and the last interpolated approximation to this value. Used in the procedure for locating a feature.
ER	Maximum permitted difference between the value of a variable defining a feature on the ray-path and the interpolated approximation to this value.
HI	Height, in km, of the current point on the ray-path. Equivalent to Q(3) (qv).
IA	DŌ-loop indexing parameter. Has value 2 unless previous hop ended at a perigee in the ionosphere when it has value 1.
II	DŌ-loop index.
J	Subscript of point at the end of a linear section of the ray-path.

JD — Control indicating the direction of the ray, ie, JD = 1, ray ascending; JD = -1, ray descending.

LB — Label of the ionospheric boundary next above the current point in the ray-hop.

Q — Array containing the current values of the dependent variables for transfer to subroutine RUKU. The first 4 elements of the array are referenced by the equivalent names; UI, VI, HI, THI (qv). Elements 5 to 7 correspond to phase, group and absorption respectively, but as they do not enter into the corresponding differential equations, they are given dummy values of zero in the DATA statement.

RE — Earth radius, 6370 km, set in the DATA statement.

STEP — Current value of the integration step length, in km, ie, the increment in ray-path length S, the independent variable.

THI — Range-angle, in radians, to the current point on the ray-path. Equivalent to Q(4) (qv).

THND — Range-angle, in radians, to the end of a ray-hop.

UI — Value of the variable $u = \mu dh/ds$ at the current point on the ray-path. Equivalent to Q(1) (qv).

UJ — Value of the variable u at the next point on the ray-path.

VI — Value of the variable $v = \mu r^2 d\theta/ds$ at the current point on the ray-path. Equivalent to Q(2) (qv).

XA, XB — Values of step-length used to compute the point immediately before and the point immediately after a feature on the ray-path. Used in the procedure for locating a feature.

Y, YA, YB, YC — Values of the variable used to locate a feature on the ray-path. They are, respectively, the value defining the feature, the values at the point before and the point after the feature, and the interpolated approximation to the required value.

<u>Blank COMMON</u>

The array C is not used. Only elements 2 and 15 from array IC, and elements 9 and 10 from array CF are required and are referenced by the following equivalent names:-

L — Label of the ionospheric boundary below the current point in the ray-hop, also the label of the ionospheric layer in which the current point is located. Equivalent to IC(15).

NB — Number of boundaries in the ionospheric model. Equivalent to IC(2).

UNGP Factor to convert group path to required units, ie, unity for group path in km, or i/c (c in km/ms) for group time in ms. Equivalent to CF(10).

UNPH Factor to convert phase path to required units, ie, unity for phase path in km, 1/c (c in km/ms) for phase time in ms, or f/c (f in MHz, c in km/μs) for phase in cycles. Equivalent to CF(9).

All other items in the list are used.

COMMON/HOP/

All items in the list (qv) with the exception of ALPHA, ALPHD and LA, are used.

Note: The variable NDR has one of the definitions given in the list description (qv) on exit from the subroutine, but has a different (local)use during the execution of the subroutine, where it acts as a control to indicate the presence or absence of a feature between two consecutive points on the ray-path and can assume the following values:-

1 No feature encountered.

2 Apogee passed.

3 Perigee passed.

4 Maximum height passed.

5 Maximum range passed.

6 Ionospheric boundary crossed on ascent.

7 Ionospheric boundary crossed on descent.

COMMON/RAYLIN/

All items in the list (qv) are used.

```
      SUBROUTINE RAY2B
C     TRACES ONE HOP OF A RAY PATH THROUGH A TWO-DIMENSIONAL IONOSPHERE
C     VERSION A (REVISION 23/10/69 (A.R.C.) OF RAY2A WRITTEN NOVEMBER 67
C                A.R.C., MODIFIED SEPTEMBER 68 K.B.B.)
C
      COMMON C(1000),IC(15),CF(10),HM,THM,ST(16)
      COMMON /HOP/ ALPHR,NDR,LA,LH,I,H(1000),THETA(1000),RHO(1000),S,
     1             PH(1000),GP(1000),AB(1000),ALPHA,ALPHD,IPK,HIPK,
     2             RHOIPK,HND,RHOND,PHND,GPND,ABND
      COMMON /RAYLIN/ N,H1,THETA1,HJ,DTH,DS,U,V
      DIMENSION Q(7),DQ(7)
      EQUIVALENCE (IC(2),NB),(IC(15),L),(CF(9),UNPH),(CF(10),UNGP),
```

```
     1            (Q(1),UI),(Q(2),VI),(Q(3),HI),(Q(4),THI),(DQ(1),DU),
     2            (DQ(2),DV),(DQ(3),DH),(DQ(4),DTHET),(DQ(5),DPH),
     3            (DQ(6),DGP),(DQ(7),DAB)
      DATA RE(6370.),(Q(K),K=5,7)(/3/0.)
C     SET DIRECTION INDEX JD (1 FOR ASCENT). INITIALISE POINT COUNT I
      JD=1
      I=1
C     CHECK FOR FIRST HOP
      IF(LH-1)54,1,2
C     INITIAL CONDITIONS FOR FIRST HOP
    1 S=0.
      H(1)=0.
      THETA(1)=0.
      RHO(1)=0.
      PH(1)=0.
      GP(1)=0.
      AB(1)=0.
      L=1
      LB=2
      GO TO 4
C     INITIAL CONDITIONS FOR A HOP OTHER THAN THE FIRST
    2 H(1)=HND
      THETA(1)=THND
      RHO(1)=RHOND
      PH(1)=PHND
      GP(1)=GPND
      AB(1)=ABND
C     IF LAST HOP ENDED AT A PERIGEE IN IONOSPHERE (NDR=3), NO LINEAR
C     SECTION IS REQUIRED. GO TO LOOP WITH INITIAL INDEX IA=1
      GO TO (4,4,3),NDR
    3 IA=1
      GO TO 10
C     SET CONTROLS AND DATA FOR LINEAR ASCENT TO IONOSPHERE BASE
    4 J=2
      IPK=J
      N=1
      H1=H(1)
      THETA1=THETA(1)
C     COMPUTE LINEAR SECTION OF RAY PATH
    5 CALL LINE2B
C     CHECK FOR MAXIMUM RANGE ON LINEAR SECTION
      THETA(J)=THETA(I)+DTH
      IF(THETA(J)-THM)7,6,6
C     MAX RANGE PASSED. RECOMPUTE LINEAR SECTION TO END RAY AT MAX RANGE
    6 DTH=THM-THETA(I)
      THETA(J)=THM
      N=N+3
      NDR=5
      CALL LINE2B
C     SET VARIABLES AT END OF LINEAR SECTION
    7 S=S+DS
      H(J)=HJ
      RHO(J)=RE*THETA(J)
      PH(J)=UNPH*DS+PH(I)
```

```
      GP(J)=UNGP*DS+GP(I)
      AB(J)=AB(I)
      I=J
C     IF N.GT.1 SET MAX AND/OR END DATA AND END RAY. OTHERWISE CONTINUE
      GO TO (9,50,8,8,50,8),N
C     END OF RAY IS MAX POINT FOR THIS HOP
    8 HIPK=H(J)
      RHOIPK=RHO(J)
      GO TO 50
C     TRANSFER U AND V TO ARRAY C (EQUIV UI,VI). SET INTEGRATION STEP
C     LENGTH FOR FIRST LAYER. INITIALISE LOOP INDEX IA=2
    9 UI=U
      VI=V
      STEP=ST(1)
      IA=2
C     CONTINUE TRACING OF RAY PATH WITH RUNGE-KUTTA ROUTINE UP TO A
C     MAXIMUM OF 1000 POINTS
   10 DO 46 II=IA,998
C     SET FEATURE INDICATOR (NDR) TO NORMAL, I.E. 1. TRANSFER CURRENT
C     VALUES OF HEIGHT AND RANGE ANGLE TO ARRAY C (EQUIV HI,THI)
      NDR=1
      HI=H(II)
      THI=THETA(II)
C     FIND INCREMENTS (DC) IN VARIABLES FOR NEXT POINT
      CALL RUKU(7,0,DC,STEP)
C     CHECK FOR MAXIMUM RANGE
      THETA(II+1)=THETA(II)+DTHET
      IF(THETA(II+1)-THM)12,11,11
C     MAXIMUM RANGE EXCEEDED. SET DATA TO LOCATE
   11 NDR=5
      YA=THETA(II)
      YC=THETA(II+1)
      Y=THM
      GO TO 23
C     SET NEXT VALUES OF U AND H AND DETERMINE RAY DIRECTION
   12 UJ=UI+DU
      H(II+1)=H(II)+DH
   13 IF(JD)20,54,14
C     RAY ASCENDING
C     CHECK FOR APOGEE. I.E. HAS U CHANGED FROM +VE TO -VE
   14 IF(UJ)15,15,16
C     APOGEE PASSED. SET CONTROLS FOR LOCATION
   15 IPK=II+1
      NDR=2
      GO TO 22
C     CHECK FOR UPPER BOUNDARY CROSSING
   16 BH=HFTH(LB,THETA(II+1))
      IF(BH-H(II+1))17,17,18
C     UPPER BOUNDARY CROSSED. SET FEATURE INDICATOR NDR=6
   17 NDR=6
      GO TO 26
C     CHECK FOR MAXIMUM HEIGHT
   18 IF(HM-H(II+1))19,19,40
C     MAXIMUM HEIGHT EXCEEDED. SET FEATURE INDICATOR NDR=4, DATUM Y=HM
```

```
   19 NDR=4
      Y=HM
      GO TO 27
C     RAY DESCENDING
C     CHECK FOR PERIGEE. I.E. HAS U CHANGED FROM -VE TO +VE
   20 IF(UJ)24,21,21
C     PERIGEE PASSED. SET FEATURE INDICATOR NDR=3
   21 NDR=3
C     DATA FOR LOCATING APOGEE OR PERIGEE (AND ER FOR MAX RANGE ANGLE)
   22 YA=UI
      YC=UJ
      Y=0.
   23 ER=.1E-9
      GO TO 28
C     CHECK FOR LOWER BOUNDARY CROSSING
   24 BH=HFTH(L,THETA(II+1))
      IF(BH-H(II+1))40,25,25
C     LOWER BOUNDARY CROSSED. SET FEATURE INDICATOR NDR=7
   25 NDR=7
C     DATUM FOR BOUNDARY CROSSING Y=BH
   26 Y=BH
C     DATA FOR LOCATING A BOUNDARY OR MAXIMUM HEIGHT
   27 YA=H(II)
      YC=H(II+1)
      ER=.1E-5
C     BY LINEAR INTERPOLATION ESTIMATE STEP NECESSARY TO LOCATE FEATURE
   28 DYA=Y-YA
      XA=0.
   29 YB=YC
      XB=STEP
   30 STEP=DYA*(XB-XA)/(YB-YA)+XA
C     RECOMPUTE POINT II+1 WITH REVISED STEP LENGTH
      CALL RUKU(7,0,DO,STEP)
      UJ=UI+DU
      H(II+1)=H(II)+DH
      THETA(II+1)=THETA(II)+DTHET
C     RESET Y DATA TO CHECK LOCATION OF FEATURE
      GO TO (54,31,31,35,32,33,34),NDR
   31 YC=UJ
      GO TO 36
   32 YC=THETA(II+1)
      GO TO 36
   33 Y=HFTH(LB,THETA(II+1))
      GO TO 35
   34 Y=HFTH(L,THETA(II+1))
   35 YC=H(II+1)
   36 DYC=Y-YC
C     CHECK ACCURACY OF LOCATION
      IF(ABS(DYC)-ER)39,39,37
C     FEATURE NOT LOCATED WELL ENOUGH. REPEAT ESTIMATION WITH POINT C
C     IN PLACE OF POINT A (DYA*DYC +VE) OR POINT B (DYA*DYC -VE)
   37 IF(DYA*DYC)29,39,38
   38 XA=STEP
```

```
      YA=YC
      DYA=DYC
      GO TO 30
C     CHECK FOR OTHER FEATURES IF NECESSARY
   39 GO TO (54,16,24,40,15,18,40),NDR
C     POINT II+1 CONFIRMED. SET REMAINING DATA
   40 S=S+STEP
      UI=UJ
      VI=VI+DV
      RHO(II+1)=RE*THETA(II+1)
      PH(II+1)=UNPH*DPH+PH(II)
      GP(II+1)=UNGP*DGP+GP(II)
      AB(II+1)=.0461*DAB+AB(II)
      I=II+1
      GO TO (45,41,49,48,47,42,43),NDR
C     APOGEE LOCATED. SET HEIGHT AND RANGE. CHANGE DIRECTION INDICATOR
C     JD TO -1 FOR DESCENT
   41 HIPK=H(II+1)
      RHOIPK=RHO(II+1)
      JD=-1
C     N.B. FIRST POINT AFTER APOGEE IS COMPUTED WITH REVISED STEP
      GO TO 46
C     UPPER BOUNDARY CROSSING LOCATED. CHECK IF TOP OF IONOSPHERE
   42 IF(LB-NB)44,51,54
C     LOWER BOUNDARY CROSSING LOCATED. CHECK IF BASE OF IONOSPHERE
   43 IF(L-1)54,52,44
C     LOCATED BOUNDARY IS WITHIN IONOSPHERE. RESET LABELS FOR NEXT LAYER
   44 L=L+JD
      LB=LB+JD
C     RESET OR CONFIRM STEP LENGTH
   45 STEP=ST(L)
C     RECYCLE FOR NEXT POINT
   46 CONTINUE
C     NORMAL EXIT FROM LOOP INDICATES RAY DATA STORES ALMOST FULL
C     RESET NDR=6 AND END THE RAY
      NDR=6
C     MAX RANGE REACHED OR STORE NEAR FULL. CHECK RAY DIRECTION
   47 IF(JD)49,54,48
C     HIGHEST POINT IS AT END OF RAY WHEN MAX HEIGHT REACHED AND
C     POSSIBLY WHEN MAX RANGE REACHED OR STORE NEAR FULL
   48 IPK=I
      HIPK=H(I)
      RHOIPK=RHO(I)
C     FIND RAY END ANGLE (MAX HEIGHT OR RANGE, STORE FULL OR PERIGEE)
   49 ALPHR=ATAN((RE+H(I))*UI/VI)
C     SET RAY END POINT DATA
   50 HND=H(I)
      THND=THETA(I)
      RHOND=RHO(I)
      PHND=PH(I)
      GPND=GP(I)
      ABND=AB(I)
      RETURN
C     TOP OF IONOSPHERE REACHED. CONTINUE RAY LINEARLY TO MAXIMUM HEIGHT
```

```
   51 J=I+1
      IPK=J
      H(J)=HM
      HJ=H(J)
      NDR=4
      N=3
      GO TO 53
C     BASE OF IONOSPHERE REACHED. CONTINUE RAY LINEARLY TO END OF HOP
   52 J=I+1
      N=2
   53 H1=H(I)
      U=UI
      V=VI
      GO TO 5
C     ERROR IN A CONTROL OR INDEX
   54 PRINT 4101
 4101 FORMAT(50H1A CONTROL OR INDEX HAS AN INVALID VALUE SOMEWHERE/////)
      STOP
      END
```

6.1.4 Subroutine LINE2B

This subroutine computes the end point data of a linear section of a ray-path. Six types of section are available, the one required being selected by the control N (see COMMON/RAYLIN/). These sections are as follows:-

1. Ray ascending to the ionosphere base from a point, height h_1, range angle θ_1, with an elevation angle α. In the present program, which only allows a ground-based transmitter, either h_1 will be zero (first hop, or later hop starting from ground reflection), α will be zero (hop starting from a perigee), or both will be zero (tangent ray). However, the more generalised form given here will be suitable for use in a program accomodating an elevated transmitter (below the ionosphere). This section is also complicated by allowing for "eccentric" ionospheric models, ie, models whose boundaries, while circular, are not concentric with the earth. The relevant geometry is given in figure 1.

For this section, the known data are h_1, θ_1 and α, together with the coordinates (r_o, θ_o) of the centre of the ionospheric model boundary system and the square of its base radius r_b (located in C(1), C(2) and C(3) respectively, see blank COMMON). The data required are the length of the linear section δs, the increment in range angle over the section $\delta\theta$, the height h_j and the values of $u = \mu dh/ds$ and $v = \mu r_j^2 d\theta/ds$ ($\mu = 1$) at the end of the section. These data are given by:-

$$v = r \cos \alpha \qquad \text{where} \qquad r = r_e + h_1,$$

$$r_e = \text{earth radius, 6370 km.}$$

$$\delta s = p_2 - r \sin \alpha \qquad \text{where} \qquad p_2 = \sqrt{r_b^2 - p_1^2} - r_o \sin \phi,$$

$$p_1 = v + r_o \cos \phi,$$

$$\phi = \alpha + \theta_o - \theta_1 .$$

$$\delta\theta = \tan^{-1}\left(\frac{\delta s \cos \alpha}{r + \delta s \sin \alpha}\right)$$

$$h_j = r_j - r_e \qquad \text{where} \qquad r_j = \sqrt{r^2 + \delta s\,(p_2 + r \sin \alpha)}$$

$$u = \frac{p_2}{r_j} .$$

2. Ray descending from the base of the ionospheric model towards the earth surface. Two possibilities arise here; either the ray reaches the earth surface (see figure 2a) or it reaches a perigee (see figure 2b).

Known data are height h_1 at exit from the ionosphere and the angle of exit (contained in the values of u and v). The required data are δs, $\delta\theta$ and h_j as defined in section 1, and the ray elevation angle at the end of the hop, α_E.

For a ray reaching the surface ($v < r_e$), the relevant equations are:-

$$h_j = 0$$

$$\alpha_E = \tan^{-1}\left(\frac{p}{v}\right) \qquad \text{where} \qquad p = \sqrt{r_e^2 - v^2}$$

$$\delta s = \frac{h_1(r + r_e)}{p - ur}$$

$$\delta\theta = \tan^{-1}\left(\frac{v\delta s}{r_e^2 + p\delta s}\right).$$

For a ray reaching a perigee ($v > r_e$), the equations are:-

$$\alpha_E = 0$$

$$h_j = v - r_e$$

$$\delta s = -\frac{u}{r}$$

$$\delta\theta = \tan^{-1}\left(\frac{\delta s}{v}\right).$$

3. Ray ascending from the top of the ionospheric model to some maximum height, eg, satellite height (see figure 3).

The known data are the height at the top of the ionosphere h_1, the angle of exit (contained in the values of u and v), and the maximum height h_j. The required data are α_E, δs and $\delta\theta$ and are given by:-

$$\alpha_E = \tan^{-1}\frac{p}{v} \qquad \text{where} \qquad p = \sqrt{r_j^2 - v^2}$$

$$\delta s = p - ur$$

$$\delta\theta = \tan^{-1}\left(\frac{v\delta s}{r(r + u\delta s)}\right).$$

The next three sections correspond with the first three but with the ray terminated earlier at some maximum range.

4. Ray ascending towards the base of the ionospheric model but stopped at maximum range (see figure 4). The known data are the height h_1 and elevation angle α at the start of the section, and the range angle increment $\delta\theta$ over the section. The output data are α_E, h_j and δs and are given by:-

$$\alpha_E = \alpha + \delta\theta$$

$$\delta s = p \sin \delta\theta \qquad \text{where} \qquad p = r \sec \alpha_E$$

$$h_j = p \cos \alpha - r_e.$$

5. Ray descending from the base of the ionospheric model to a point at maximum range (see figure 5). The data α_E, δs and h_j are derived from the known values of $\delta\theta$, h_1, u and v as follows:-

$$\alpha_E = \delta\theta + \tan^{-1}\left(\frac{ur}{v}\right)$$

$$\delta s = r \sin \delta\theta \sec \alpha_E$$

$$h_j = v \sec \alpha_E - r_e.$$

6. Ray ascending from the top of the ionospheric model to a point at maximum range (see figure 6). The required data are obtained as in section 5.

STORAGE LIST

Local Variables

CA	Cosine of ALPHR (see COMMON/HOP/).
P	An intermediate variable, various definitions.
PHI	An intermediate angle, in radians.
R	Radius, in km, to start of linear section of ray-path.
RE	Earth radius, 6370 km, set in DATA statement.
RESQ	Square of earth radius, set in DATA statement.
RJ	Radius, in km, to end of linear section of ray-path.
RJSQ	Square of RJ (above).
RSA	Product of R (above) and sine of ALPHR (see COMMON/HOP/).
SA	Sine of ALPHR (see COMMON/HOP/).
SECA	Secant of ALPHR (see COMMON/HOP/).
TWORE	Earth diameter, ie, twice RE (above), set in DATA statement.
VSQ	Square of V (see COMMON/RAYLIN/).

Blank COMMON

C	The first three elements of this array are the only items in the list (qv) used here, and are referenced by the local names C1, C2 and C3 respectively.

COMMON/HOP/

The first two items (ALPHR, NDR) in the list (qv) are used here.

COMMON/RAYLIN/

The full list (qv) is required.

```
      SUBROUTINE LINE2B
C     CALCULATES END POINT DATA OF LINEAR SECTIONS OF RAY PATHS
C     VERSION A (WRITTEN A.R.C. REVISED 21/10/69)
C
      COMMON C1,C2,C3
      COMMON /HOP/ ALPHR,NDR
      COMMON /RAYLIN/ N,H1,THETA1,HJ,DTH,DS,U,V
      DATA RE(6370.),RESQ(40576900.),TWORE(12740.)
C
C     SET RADIUS OF INITIAL POINT AND SELECT REQUIRED SECTION
```

```
      R=H1+RE
      GO TO (10,20,30,40,50,50),N
C
C     SECTION 1. ASCENT TO BASE OF IONOSPHERE (N=1)
C
   10 SA=SIN(ALPHR)
      RSA=SA*R
      CA=COS(ALPHR)
      V=CA*R
      PHI=ALPHR+C2-THETA1
      P=COS(PHI)*C1+V
      P=SQRT(C3-P*P)-C1*SIN(PHI)
      DS=P-RSA
      DTH=ATAN(CA*DS/(DS*SA+R))
      RJ=SQRT((P+RSA)*DS+R*R)
      HJ=RJ-RE
      U=P/RJ
      RETURN
C
C     SECTION 2. DESCENT FROM IONOSPHERE BASE (N=2). TWO ALTERNATIVES
C     ARE POSSIBLE, EITHER THE RAY LANDS OR IT REACHES A PERIGEE
C
   20 HJ=V-RE
      IF(HJ)21,21,22
C     RAY LANDS
   21 NDR=1
      P=SQRT(-(RE+V)*HJ)
      ALPHR=ATAN(P/V)
      DS=(R+RE)*H1/(P-U*R)
      DTH=ATAN(V*DS/(DS*P+RESQ))
      HJ=0.
      RETURN
C     STRAIGHT PERIGEE
   22 NDR=2
      ALPHR=0.
      DS=-U*R
      DTH=ATAN(DS/V)
      RETURN
C
C     SECTION 3. ASCENT FROM TOP OF IONOSPHERE TO MAXIMUM HEIGHT (N=3)
C
   30 VSQ=V*V
      RJSQ=(TWORE+HJ)*HJ+RESQ
      P=SQRT(RJSQ-VSQ)
      ALPHR=ATAN(P/V)
      DS=P-U*R
      DTH=ATAN(V*DS/((DS*U+R)*R))
      RETURN
C
C     SECTION 4. POINT AT MAX RANGE ON ASCENT TO IONOSPHERE BASE (N=4)
C
   40 CA=COS(ALPHR)
      ALPHR=ALPHR+DTH
      P=R/COS(ALPHR)
```

```
      HJ=P*CA-RE
      DS=SIN(DTH)*P
      RETURN
C
C     SECTION 5. POINT AT MAX RANGE ON DESCENT FROM BASE OF IONOSPHERE
C     (N=5) OR ON ASCENT FROM TOP OF IONOSPHERE (N=6)
C
   50 ALPHR=ATAN(U*R/V)+DTH
      SECA=1./COS(ALPHR)
      HJ=SECA*V-RE
      DS=SIN(DTH)*R*SECA
      RETURN
      END
```

6.1.5 Subroutine RUKU

This is a general purpose subroutine containing a standard Runge-Kutta fourth-order method for the solution of a set of first-order ordinary differential equations, of the form

$$\frac{dq_m}{dq_o} = F_m(q_1, q_2, \ldots\ldots q_n) \quad \text{for} \quad m = 1, n,$$

where q_o is the independent variable and the q_m are the dependent variables. Note: q_o may appear as one of the q_m, eg, as q_j, by defining

$$\frac{dq_j}{dq_o} = 1.$$

If at $q_o = Q_o$, $q_m = Q_m$ for $m = 1, n$,

then at $q_o = Q_o + \delta q_o$, $q_m = Q_m + \delta q_m$ for $m = 1, n$,

where $\delta q_m = (K_{m1} + 2K_{m2} + 2K_{m3} + K_{m4})/6$,

$$K_{m1} = \delta q_o . F_m(Q_1, Q_2, \ldots\ldots Q_n),$$

$$K_{m2} = \delta q_o . F_m(Q_1 + \tfrac{1}{2}K_{11}, Q_1 + \tfrac{1}{2}K_{21}, \ldots\ldots Q_n + \tfrac{1}{2}K_{n1}),$$

$$K_{m3} = \delta q_o . F_m(Q_1 + \tfrac{1}{2}K_{12}, Q_2 + \tfrac{1}{2}K_{22}, \ldots\ldots Q_n + \tfrac{1}{2}K_{n2})$$

and $$K_{m4} = \delta q_o . F_m(Q_1 + K_{13}, Q_2 + K_{23}, \ldots\ldots Q_n + K_{n3}).$$

The routine accepts the current values of the variables, Q_m, and returns their increments, δq_m, for an increment, δq_o, in the independent variable. The differential equations are contained in a subroutine DEQS, which, on receiving values of the variables, returns corresponding values of their derivatives. To avoid storing all the K_{mi} until the end of the routine, the δq_m are accumulated as the routine proceeds. As written, the subroutine can handle up to 20 differential equations, although the present ray-tracing program only requires 7.

STORAGE LIST

Argument Cells

DQ (output)	Array containing increments in the dependent variables.
DQO (input)	Integration step length, ie, increment in the independent variable.
N (input)	Number of dependent variables.
Q (input)	Array containing current values of the dependent variables.

Local Variables

DQI	Intermediate increment in a dependent variable.
FQ	Array containing derivatives of the dependent variables.
J	DO-loop index.
QI	Array containing intermediate values of the dependent variables.
R6	Reciprocal of 6, set in DATA statement.

```
      SUBROUTINE RUKU(N,Q,DQ,DQO)
C     STANDARD RUNGE-KUTTA FOURTH-ORDER METHOD FOR THE SOLUTION OF A SET
C     OF N ORDINARY DIFFERENTIAL EQUATIONS. (DEFINED IN SUBROUTINE DFQS)
C     THE PROGRAM EVALUATES THE INCREMENTS DQ IN THE VARIABLES Q FOR A
C     STEP DQO IN THE INDEPENDENT VARIABLE
C     VERSION A (REVISED 17/10/69 A.R.C.)
C
      DIMENSION Q(N),DQ(N),QI(20),FQ(20)
      DATA R6(.16666666666667)
C
      CALL DEQS(N,Q,FQ)
      DO 1 J=1,N
      DQ(J)=FQ(J)*DQO
      QI(J)=DQ(J)*.5+Q(J)
    1 CONTINUE
C
      CALL DEQS(N,QI,FQ)
      DO 2 J=1,N
      DQI=FQ(J)*DQO
      QI(J)=DQI*.5+Q(J)
      DQ(J)=DQ(J)+DQI+DQI
    2 CONTINUE
C
      CALL DEQS(N,QI,FQ)
      DO 3 J=1,N
      DQI=FQ(J)*DQO
      QI(J)=DQI+Q(J)
      DQ(J)=DQ(J)+DQI+DQI
```

```
      3 CONTINUE
C
        CALL DEQS(N,QI,FQ)
        DO 4 J=1,N
        DQ(J)=(FQ(J)*DQO+DQ(J))*R6
      4 CONTINUE
        RETURN
        END
```

6.1.6 Subroutine DEQS

This subroutine, containing the differential equations of the ray-path, computes the derivatives of the ray variables corresponding to the available values of the variables.

The differential equations are as follows:-

FQ(1). $\frac{du}{ds} = \frac{v^2}{\mu r^3} + \frac{\partial \mu}{\partial h}$ $(r = r_e + h)$

FQ(2). $\frac{dv}{ds} = \frac{\partial \mu}{\partial \theta}$

FQ(3). $\frac{dh}{ds} = \frac{u}{\mu}$

FQ(4). $\frac{d\theta}{ds} = \frac{v}{\mu r^2}$

FQ(5). $\frac{d}{ds}(\text{phase}) = \mu$ See note 1

FQ(6). $\frac{d}{ds}(\text{group}) = \frac{1}{\mu}$ See note 1

FQ(7). $\frac{d}{ds}(\text{absorption}) = \frac{Ne.\nu}{\mu(\omega^2 + \nu^2)}$ See note 2

Inputs to the subroutine are u, v, height h and range angle θ. The appropriate electron density Ne, refractive index μ and its derivatives $\partial\mu/\partial h$, $\partial\mu/\partial\theta$ are obtained from subroutine NRX. The collision frequency ν is obtained from the function COLF. The square of the angular frequency ω is preset in the array CF (see blank COMMON).

Note 1. Solution of the stated phase and group equations gives results in km. These results are modified in subroutine RAY2B to give outputs in the required units, ie, km, ms or (phase only) cycles.

Note 2. A constant factor of 0.0461 is omitted from the absorption equation, but is applied to the result in subroutine RAY2B.

STORAGE LIST

Argument Cells

FQ (output) — Array containing the derivatives of the variables corresponding to the values Q (below).

N (input) — Number of differential equations.

Q (input) — Array containing current values of the variables.
Here, $Q(1) = u = \mu dh/ds$
$Q(2) = v = \mu r^2 d\theta/ds$
$Q(3) = h$
$Q(4) = \theta$
Q(5), Q(6), Q(7) correspond to phase, group and absorption, but the values are not required in the differential equations.

Local Variables

ECF — Electron collision frequency, number per second.

RR — Reciprocal of the radial distance, $r_e + h$, to the current point.

VRR — Product of v (see Q(2) above) and RR (above).

Blank COMMON

CF — The eighth element of this array is the only item in the list (qv) used here, and is referenced locally by the equivalent name WSQ (square of angular frequency).

COMMON/PT/

All items in the list (qv) up to and including DMUDTH are required here.

```
      SUBROUTINE DEQS(N,Q,FQ)
C     VERSION A (REVISED 16/10/69 A.R.C.)
C     DIFFERENTIAL EQUATIONS OF 2-DIMENSIONAL RAY PATH (1 TO 4),
C     PHASE (5) AND GROUP (6) PATHS AND ABSORPTION (7)
      COMMON C(1000),IC(15),CF(10)
      COMMON /PT/ H,THETA,FN,FMU,RMU,DMUDH,DMUDTH
      DIMENSION Q(N),FQ(N)
      EQUIVALENCE (CF(8),WSQ)
C     SEND HEIGHT AND RANGE ANGLE TO COMMON /PT/ LIST
      H=Q(3)
      THETA=Q(4)
C     F R' RECIPROCAL (RR) OF R (I.E. OF RE+H) AND QUOTIENT (VRR) V/R
      RR=1./(6370.+H)
      VRR=RR*Q(2)
C     FIND COLLISION FREQUENCY (ECF) FOR ABSORPTION EQUATION
```

```
      ECF=COLF(H)
C     FIND ELECTRON DENSITY (EN) AND REFRACTIVE INDEX (EMU), ITS
C     RECIPROCAL (RMU) AND DERIVATIVES (DMUDH, DMUDTH)
      CALL NRX
C     FIND DERIVATIVES AT CURRENT VALUES OF VARIABLES
C     RAY EQUATIONS
      FO(2)=DMUDTH
      FO(3)=O(1)*RMU
      FO(4)=VRR*RR*RMU
      FO(1)=FO(4)*VRR+DMUDH
C     PHASE AND GROUP EQUATIONS. SOLUTIONS OF THE FORMS BELOW ARE IN KM.
C     UNITS ARE REVISED, IF REQUIRED, IN SUBROUTINE RAY2R
      FO(5)=EMU
      FO(6)=RMU
C     ABSORPTION EQUATION. N.B. CONSTANT FACTOR OF .0461 IS OMITTED FROM
C     THE FORM BELOW BUT IS APPLIED TO SOLUTION IN SUBROUTINE RAY2R
      FO(7)=EN*ECF*RMU/(ECF*ECF+WSO)
      RETURN
      END
```

6.1.7 Function COLF

This function computes the electron collision frequency at a given height from a suitable model.

The model in this version of the function consists of the sum of two exponential terms, viz,

$$\nu = 3.65 \times 10^{11} e^{-0.158h} + 2.08 \times 10^{3} e^{-0.00424h}.$$

The first term, which makes the major contribution at heights below about 120 km where collisions are most effective, is a fair fit to data available in the literature, eg, [5]. The second term, predominant above 125 km where the effects of collisions are much smaller, offers no more than a possible trend in collision frequency at such heights.

A graph of the model is shown in figure 7.

STORAGE LIST

Argument Cells

COLF (output) Electron collision frequency, ν, number per second.

H (input) Height, h, km.

```
      FUNCTION COLF(H)
C     COLLISION FREQUENCY FUNCTION
C     VERSION A (REVISED 21/10/69 A.R.C.)
C     TWO EXPONENTIAL SECTIONS. FIRST IS A FIT TO DATA IN THE LITERATURE
C     TO ABOUT E-LAYER PEAK HEIGHTS. SECOND GIVES ORDER OF MAGNITUDE
C     ESTIMATES AT GREATER HEIGHTS. (FIRST IS NEGLECTED ABOVE 300 KM.)
      IF(H-300.)1,2,2
    1 COLF=.365E+12*EXP(-.158*H)+.208E+04*EXP(-.00424*H)
      RETURN
    2 COLF=.208E+04*EXP(-.00424*H)
      RETURN
      END
```

6.2 Group (ii) subroutines

6.2.1 Subroutine PRNTPT.

6.2.2 Subroutine RYPRNT.

6.2.3 Subroutine RYPNCH.

6.2.4 Subroutine EXPHGP.

6.2.5 Subroutine TŌTSIG.

6.2.6 Subroutine DATAX.

6.2.1 Subroutine PRNTPT

This subroutine enables the printing of the height, range, phase and absorption data for each point in a ray-hop. Each page contains the data for 80 points and is headed with identification and summary data for the particular hop.

STORAGE LIST

Local Variables

M Loop index for printing data for each point.

MA Subscript of first point to be printed on a page.

MB Subscript of last point to be printed on a page.

MP DO-loop index for pages of printout.

NP Number of pages required for printing data for one ray-hop.

COMMON/IMO/

The first four items in the list (qv) are required.

COMMON/HOP/

All items in the list (qv) up to and including IPK are used with the exception of ALPHR, LA and THETA.

```
      SUBROUTINE PRNTPT
C     PRINTS OUT DATA FOR EACH COMPUTED POINT IN A RAY-HOP
C     VERSION A (REVISION AND EXTENSION 4/11/69 A.R.C. OF SUB DPRINT
C                14/2/69 K.B.B.)
C
      COMMON /IMO/ JOBNO,IID,LT,F
      COMMON /HOP/ ALPHR,NDR,LA,LH,I,H(1000),THETA(1000),RHO(1000),S,
     1             PH(1000),GP(1000),AB(1000),ALPHA,ALPHD,IPK
C
C     FIND NUMBER OF PAGES REQUIRED AT 80 DATA POINTS PER PAGE
      NP=I/80
      IF(NP*80.LT.I)NP=NP+1
C     INITIALISE SUBSCRIPT OF LAST POINT ON A PAGE
      MB=0
C     EXECUTE THE FOLLOWING LOOP FOR THE RELEVANT NUMBER OF PAGES
      DO 1 MP=1,NP
C     SET SUBSCRIPTS OF FIRST AND LAST POINTS ON PAGE
      MA=MB+1
      MB=MA+79
      IF(MB.GT.I)MB=I
C     TITLE A NEW PAGE
      PRINT 4101,JOBNO,IID,LT,F,ALPHA,LH,ALPHD,S,I,IPK,NDR
C     PRINT DATA FOR THIS PAGE
      PRINT 4102,(M,H(M),RHO(M),PH(M),GP(M),AB(M),M=MA,MB)
C     NEXT PAGE
    1 CONTINUE
      RETURN
C     PRINTOUT FORMATS
 4101 FORMAT(4H1JOB,I5,5H  IID,I4,1H/,I2,4H  F=,F7.3,11HMHZ  ALPHA=,F6.3
     1,8HDEG  HOP,I2,8H  ALPHD=,F7.3,10HDEG  PATH=,F7.1,2HKM,I6,10H PTS
     2 IPK=,I3,6H  NDR=,I2//117H  I    HEIGHT    RANGE      PHASE
     3  GROUP       ABS     I    HEIGHT    RANGE      PHASE         GROU
     4P       ABS/)
 4102 FORMAT(2(I4,F9.3,F10.3,2E14.8,F7.3,2X))
      END
```

6.2.2 Subroutine RYPRNT

This subroutine prints out the salient data for each ray-hop, one line of data per hop. The data include apogee height and range, end point range, phase, group and absorption, point count, etc.

A system of line counting for page control is included and identification data, eg, job, ionospheric model, are printed at the head of each page.

STORAGE LIST

Local Variables

LA — DŌ-loop index for takeoff angles (ie, for rays).

LH — DŌ-loop index for hops.

LIN — Line counter. When the line count exceeds 35 at the start of a pass through the angle loop a fresh page of printout is initiated.

PGU — Array containing the units of phase and group data for column heading. The three elements of the array contain (in A mode):- (KM), (MSEC) and (CYCLES) respectively. (The latter applies to phase only.)

CŌMMŌN/IMŌ/

The list (qv) up to and including KG is required, but ALPH1 and DA are not used.

CŌMMŌN/ŌP/

All items in the list (qv) up to and including RHŌK are used.

```
      SUBROUTINE RYPRNT
C     PRINTS OUT SUMMARY OF DATA FOR EACH RAY HOP
C     VERSION A (REVISION 24/10/69 A.R.C. OF SUB RPRINT 14/2/69 K.B.B.)
C
      COMMON /IMO/ JOBNO,IID,LT,F,NA,NH,DA,ALPH1,KP,KG
      COMMON /OP/ KEY(5,100),NDRL(5,100),RHOL(5,100),ALPHAS(100),
     1            PHL(5,100),GPL(5,100),ABL(5,100),HPK(5,100),SL(5,100),
     2            HL(5,100),ALPHDL(5,100),IL(5,100),RHOK(5,100)
C
C     COLUMN HEADINGS FOR PHASE AND GROUP DATA
      DIMENSION PGU(3)
      DATA PGU(1)(8H  (KM)  ),PGU(2)(8H (MSEC) ),PGU(3)(8H(CYCLES))
C     INITIALISE LINE COUNT TO FORCE A NEW PAGE
      LIN=36
C     START OF ANGLE (RAY) LOOP
      DO 6 LA=1,NA
C     CHECK LINE COUNT
      IF(LIN-35)2,2,1
C     TITLE A NEW PAGE AND RESTART LINE COUNT
    1 PRINT 401,JOBNO,IID,PGU(KP),PGU(KG)
      LIN=4
C     START OF HOP LOOP
    2 DO 4 LH=1,NH
C     CHECK THAT THIS HOP EXISTS
      IF(KEY(LH,LA))5,5,3
C     PRINT DATA FOR THIS HOP AND ADVANCE LINE COUNT
    3 PRINT 402,LT,F,ALPHAS(LA),ALPHDL(LH,LA),LH,HPK(LH,LA),RHOK(LH,LA),
```

```
     1          HL(LH,LA),RHOL(LH,LA),SL(LH,LA),PHL(LH,LA),GPL(LH,LA),
     2          ABL(LH,LA),IL(LH,LA),NDRL(LH,LA)
       LIN=LIN+1
C      NEXT HOP
     4 CONTINUE
C      PRINT A BLANK LINE AFTER DATA FOR EACH RAY AND ADVANCE LINE COUNT
     5 PRINT 403
       LIN=LIN+1
C      NEXT ANGLE (RAY)
     6 CONTINUE
       RETURN
C      FORMATS FOR PRINTOUT
   401 FORMAT(8H1    JOB,I5,7X16HIONOSPHERE MODEL,I4//83H LT  FREQ  ALPHA
     1   ALPHD  H   MAX-HT  RANGE-HMX  END-HT   END-RANGE   PATH   PHAS
     2E,A8,7H  GROUP,A8,14H  ABS-DB PTS N/)
   402 FORMAT(1H ,I2,F7.3,F6.2,F9.4,I2,F10.4,F10.3,F10.4,F11.4,F8.1,
     1       2E15.9,F8.4,I4,I2)
   403 FORMAT(1H )
       END
```

6.2.3 Subroutine RYPNCH

This subroutine enables the punching of significant data for each ray-hop on "RAYSET" cards. Such cards were termed RAYSETS by Croft [4] who originated the idea of storing computed ray data on punched cards for use in other programs.

Two cards are punched for each ray-hop, the first holding identification data (eg, job number, ionospheric model number, frequency, etc) and ray geometry data (eg, maximum height reached, hop range, landing angle, etc), while the second holds similar identification data together with phase, group and absorption data.

Provision is made for the output for a set of rays to be ordered in one of two ways. Either the punching is cycled by hops, ie, hop 1 - ray 1, ray 2, ... ray N; hop 2 - ray 1, ray 2, ... ray N; etc, or it is cycled by rays, ie, ray 1 - hop 1, hop 2, ... hop M; ray 2 - hop 1, hop 2, ... hop M; etc.

Header and trailer cards are punched for each set of rays to facilitate identification of the sections of the output deck which may contain additional output (see subroutine DATAX).

STORAGE LIST

Local Variables

LA — DO-loop index for angles (ie, rays).

LH — DO-loop index for hops.

X — An A8 word consisting of 8 asterisks, used on header and trailer cards of a batch of output as an aid to identification. Set in DATA statement.

COMMON/IMO/

The following items in the list (qv) are used here: F, IID, JOBNO, JP, LT, NA and NH.

COMMON/OP/

All items in the list (qv) up to and including IL are used here.

```
      SUBROUTINE RYPNCH
C     PUNCHES SUMMARY OF DATA FOR EACH RAY-HOP ON *RAYSET* CARDS
C     TWO CARDS ARE PRODUCED FOR EACH RAY-HOP
C     VERSION A (CORRECTION AND REVISION 31/10/69 A.R.C. OF SUB PUNRAY
C                LAST MODIFIED 14/2/69 K.B.B.)
C
C     FIRST CARD IN A PAIR CONTAINS (A) JOB NO. (B) IONOSPHERE MODEL NO.
C     (C) TIME-STEP NO. (D) FREQUENCY (E) RAY TAKEOFF ANGLE (F) ANGLE AT
C     END OF HOP (EOH) (G) HOP NO. (H) MAX HEIGHT GAINED IN HOP (I) EOH
C     HEIGHT (J) RANGE TO EOH (K) PATH LENGTH TO EOH (L) NO. OF POINTS
C     IN HOP (M) EOH INDICATOR
C     SECOND CARD IN A PAIR REPEATS ITEMS (A) TO (E), (G) AND (J), THEN
C     (N) PHASE TO EOH (O) GROUP TO EOH (P) ABSORPTION TO EOH (M) ABOVE
C     HEADER AND TRAILER CARDS ARE PUNCHED FOR EACH TIME-STEP/FREQUENCY
C     SET
C
      COMMON /IMO/ JOBNO,IID,LT,F,NA,NH,DA,ALPH1,KP,KG,KX,JP
      COMMON /OP/ KEY(5,100),NDRL(5,100),RHOL(5,100),ALPHAS(100),
     1            PHL(5,100),GPL(5,100),ABL(5,100),HPK(5,100),SL(5,100),
     2            HL(5,100),ALPHDL(5,100),IL(5,100)
      DATA X(8H********)
C
C     PUNCH HEADER CARD FOR THIS BATCH
      PUNCH 4201,F,F,F,F,F,X,X,X,X,X
C     SELECT ORDER OF PUNCHING
      GO TO (1,5),JP
C     PUNCH CARDS FOR ALL RAYS FOR EACH HOP IN TURN
    1 DO 4 LH=1,NH
      DO 3 LA=1,NA
C     CHECK THAT RAY-HOP EXISTS
      IF(KEY(LH,LA))3,3,2
C     PUNCH *RAYSETS* FOR THIS RAY-HOP
```

```
    2 PUNCH 4202,JOBNO,IID,LT,F,ALPHAS(LA),ALPHDL(LH,LA),LH,HPK(LH,LA),
     1            HL(LH,LA),RHOL(LH,LA),SL(LH,LA),IL(LH,LA),NDRL(LH,LA)
      PUNCH 4203,JOBNO,IID,LT,F,ALPHAS(LA),LH,RHOL(LH,LA),PHL(LH,LA),
     1            GPL(LH,LA),ABL(LH,LA),NDRL(LH,LA)
    3 CONTINUE
    4 CONTINUE
      GO TO 9
C     PUNCH CARDS FOR ALL HOPS OF EACH RAY IN TURN
    5 DO 8 LA=1,NA
      DO 7 LH=1,NH
C     CHECK THAT RAY-HOP EXISTS
      IF(KEY(LH,LA))8,8,6
C     PUNCH *RAYSETS* FOR THIS RAY-HOP
    6 PUNCH 4202,JOBNO,IID,LT,F,ALPHAS(LA),ALPHDL(LH,LA),LH,HPK(LH,LA),
     1            HL(LH,LA),RHOL(LH,LA),SL(LH,LA),IL(LH,LA),NDRL(LH,LA)
      PUNCH 4203,JOBNO,IID,LT,F,ALPHAS(LA),LH,RHOL(LH,LA),PHL(LH,LA),
     1            GPL(LH,LA),ABL(LH,LA),NDRL(LH,LA)
    7 CONTINUE
    8 CONTINUE
C     PUNCH TRAILER CARD FOR THIS BATCH
    9 PUNCH 4204,X,X,X,X,X,F,F,F,F,F
      RETURN
C     PUNCHING FORMATS
 4201 FORMAT(5F8.3,5A8)
 4202 FORMAT(2I4,I3,2F7.3,F8.4,I2,2F10.4,F11.4,F8.1,I4,I2)
 4203 FORMAT(2I4,I3,2F7.3,I2,F11.4,2E15.9,F9.4,I3)
 4204 FORMAT(5A8,5F8.3)
      END
```

6.2.4 Subroutine EXPHGP

This subroutine is used for the calculation and printout of the values of excess phase and group data at the end of each ray-hop. These excess values are the amounts by which the total values, obtained by integration along the rays, exceed equivalent values for a free space path equal to the great circle range to the end of the relevant ray-hop. The units of the excess data may be in km, μs or (phase only) cycles, as required.

A system of line counting for page control of the printout is included, and identification data, eg, job, ionospheric model, etc, are printed at the head of each page.

STORAGE LIST

Local Variables

CFG — Factor to convert excess group path data to required units, ie, value unity for excess group path in km, or value 1/c (c = velocity of light, km/μs) for excess group time in μs.

CFP — Factor to convert excess phase path data to required units, ie, value unity for excess phase path in km, value 1/c (c = velocity of light, km/μs) for excess phase time in μs or value f/c (f = frequency, MHz) for excess phase in cycles.

LA — DŌ-loop index for takeoff angles (ie, rays).

LG — Control to select column heading (see XPGU) for excess group data printout, ie, 1 for (KM) or 2 for (MUSECS).

LH — DŌ-loop index for hops.

LIN — Line counter. When the count exceeds 35 at the end of the printout for a ray, a new page of output is initiated.

LP — Control to select column heading (see XPGU) for excess phase data printout, ie, 1 for (KM), 2 for (MUSECS) or 3 for (CYCLES).

RCF9 — Reciprocal of CF9 (see blank CŌMMŌN list).

RCF10 — Reciprocal of CF10 (see blank CŌMMŌN list).

XPGU — Array of A-mode data forming the column headings for the printout of excess phase and group data. Contents are: (KM), (MUSECS) and (CYCLES), set in DATA statement (see LG and LP).

Blank CŌMMŌN

The only items in the list (qv) used here are elements 9 and 10 of the array CF, referenced locally by the names CF9 and CF10 respectively.

CŌMMŌN/IMŌ/

The list (qv) up to and including KX is used with the exception of ALPH1, DA, KG and KP.

CŌMMŌN/ŌP/

The following items from the list (qv) are used here: ALPHAS, GPL, KEY, NDRL, PHL, RHŌL, XGP and XPH.

```
      SUBROUTINE EXPHGP
C     CALCULATES AND PRINTS EXCESS PHASE AND GROUP DATA IN SPECIFIED
C     UNITS FOR EACH RAY HOP
C     VERSION A (REVISION 27/10/69 A.R.C. OF SUB EXCESS 14/2/69 K.B.B.)
C
      COMMON C(1000),IC(15),CF(8),CF9,CF10
      COMMON /IMO/ JOBNO,IID,LT,F,NA,NH,DA,ALPH1,KP,KG,KX
      COMMON /OP/ KEY(5,100),NDRL(5,100),RHOL(5,100),ALPHAS(100),
     1            PHL(5,100),GPL(5,100),ABL(5,100),HPK(5,100),SL(5,100),
     2            HL(5,100),ALPHDL(5,100),IL(5,100),RHOK(5,100),
     3            ALPHRS(100),ALPHRL(5,100),XPH(5,100),XGP(5,100)
C
C     COLUMN HEADINGS FOR EXCESS PHASE AND GROUP DATA
      DIMENSION XPGU(3)
      DATA XPGU(1)(8H  (KM)  ),XPGU(2)(8H(MUSECS)),XPGU(3)(8H(CYCLES))
C     FORM RECIPROCALS OF CF(9) AND CF(10) TO CONVERT PHASE AND GROUP
C     DATA BACK TO KM
      RCF9=1./CF9
      RCF10=1./CF10
C     SELECT REQUIRED UNITS OF EXCESS PHASE DATA
      GO TO (1,1,2,2,3,3),KX
C     EXCESS PHASE PATH IN KM
    1 CFP=1.
      LP=1
      GO TO 4
C     EXCESS PHASE TIME IN MICROSECONDS
    2 CFP=3.3356404846686
      LP=2
      GO TO 4
C     EXCESS PHASE IN CYCLES
    3 CFP=F*3.3356404846686
      LP=3
C     SELECT REQUIRED UNITS OF EXCESS GROUP DATA
    4 GO TO (5,6,5,6,5,6),KX
C     EXCESS GROUP PATH IN KM
    5 CFG=1.
      LG=1
      GO TO 7
C     EXCESS GROUP TIME IN MICROSECONDS
    6 CFG=3.3356404846686
      LG=2
C     INITIALISE LINE COUNT TO FORCE A NEW PAGE
    7 LIN=36
C     START OF ANGLE (RAY) LOOP
      DO 13 LA=1,NA
C     CHECK LINE COUNT
      IF(LIN-35)9,9,8
C     TITLE A NEW PAGE AND RESTART LINE COUNT
    8 PRINT 401,JOBNO,IID,LT,F,XPGU(LP),XPGU(LG)
      LIN=4
C     START OF HOP LOOP
    9 DO 11 LH=1,NH
C     CHECK THAT THIS HOP EXISTS
      IF(KEY(LH,LA))12,12,10
```

```
C     CALCULATE EXCESS PHASE AND GROUP DATA FOR THIS HOP
   10 XPH(LH,LA)=(PHL(LH,LA)*RCF9-RHOL(LH,LA))*CFP
      XGP(LH,LA)=(GPL(LH,LA)*RCF10-RHOL(LH,LA))*CFG
C     PRINT DATA FOR THIS HOP AND ADVANCE LINE COUNT
      PRINT 402,ALPHAS(LA),LH,NDRL(LH,LA),RHOL(LH,LA),XPH(LH,LA),
     1          XGP(LH,LA)
      LIN=LIN+1
C     NEXT HOP
   11 CONTINUE
C     PRINT A BLANK LINE AFTER DATA FOR EACH RAY AND ADVANCE LINE COUNT
   12 PRINT 403
      LIN=LIN+1
C     NEXT ANGLE (RAY)
   13 CONTINUE
      RETURN
C     FORMATS FOR PRINTOUT
  401 FORMAT(8H1    JOB,I5,7X16HIONOSPHERE MODEL,I4,5X9HTIME STEP,I3,
     18X11HFREQUENCY =,F7.3,4H MHZ//52H TAKEOFF ANGLE  HOP  TYPE     RAN
     2GE     EXCESS PHASE,A8,14H  EXCESS GROUP,A8/)
  402 FORMAT(1H ,F9.2,I8,I6,F13.4,F17.3,F22.3)
  403 FORMAT(1H )
      END
```

6.2.5 Subroutine TOTSIG

This subroutine calculates the signal loss due to spatial dispersion of rays and combines it with the relevant value of absorption to give an estimate of the total signal loss along a ray, excluding losses on ground reflection.

The calculation is based upon the expression, derived in reference [2], which states that, due to spatial dispersion, a 1 watt, ground-based, isotropic transmitter delivers a power flux of

$$\frac{\cos\alpha}{4\pi r_e \sin(\rho_D/r_e)\sin\alpha_D}\frac{\delta\alpha}{\delta\rho} \text{ watts per square kilometer}$$

at a point (on the ground), range ρ_D km from the transmitter. α is the takeoff angle of the ray reaching ρ_D, α_D is the ray landing angle, r_e is the earth radius (6370 km) and $\delta\rho$ is the small change in range due to a small increment $\delta\alpha$ (radians) in the takeoff angle.

The loss due to spatial dispersion is therefore

$$10\log\left(\frac{4\pi r_e \sin(\rho_D/r_e)\sin\alpha_D}{\cos\alpha}\left|\frac{\delta\rho}{\delta\alpha}\right|\right) \text{ db with}$$

respect to 1 watt per square kilometer.

The value of $\delta\rho/\delta\alpha$ is estimated, using three adjacent rays of ranges ρ_{n-1}, ρ_n and ρ_{n+1}, from one of the following forms:-

At ρ_{n-1} $\quad \left|\frac{\delta\rho}{\delta\alpha}\right| \simeq \frac{1}{2\delta\alpha}(3\rho_{n-1} - 4\rho_n + \rho_{n+1})$.

At ρ_n $\quad \left|\frac{\delta\rho}{\delta\alpha}\right| \simeq \frac{1}{2\delta\alpha}(\rho_{n-1} - \rho_{n+1})$.

At ρ_{n+1} $\quad \left|\frac{\delta\rho}{\delta\alpha}\right| \simeq \frac{1}{2\delta\alpha}(\rho_{n-1} - 4\rho_n + 3\rho_{n+1})$,

ie, $\quad \left|\frac{\delta\rho}{\delta\alpha}\right| \simeq \frac{1}{2\delta\alpha} f(\rho)$

where $f(\rho)$ is one of the three parenthesised expressions in the above approximations.

If the takeoff angle increment is expressed in degrees, the expression for the spatial dispersion loss becomes

$$10\log\left[\frac{360\, r_e}{\delta\alpha}\frac{\sin(\rho_D/r_e)\sin\alpha_D}{\cos\alpha}\right]f(\rho).$$

This is the form used in the subroutine, and the second expression for $f(\rho)$ is used where possible.

In cases where rays do not land or where there are less than three consecutive landing rays, the spatial dispersion and total losses are given dummy values of zero (this being an easy number to identify in the output).

Rays reflecting in different ionospheric layers are treated in separate batches. The present program can only handle such mode splitting at one height - normally between E and F layers, and also it can only identify the split in the hop under consideration, eg, if, in one hop, two adjacent rays reflect, one in the E and one in the F, they are treated in separate batches, but if, in the next hop, due to an ionospheric tilt, they both reflect in the same layer, they are treated in the same batch, giving results of doubtful value. However, cases like this are normally rare enough not to be a nuisance.

In the printout section of the program, a system of line counting for page control is included, and identification data, eg, job, ionospheric model, are printed at the head of each page of output.

STORAGE LIST

Local Variables

CŌNDBS	A constant loss, in db, arising in the evaluation of spatial dispersion and derived from $360r_e/\delta\alpha$ (r_e = 6370 km, $\delta\alpha$ = takeoff angle increment in degrees). See notes on subroutine.
FRHŌ	A function of the ranges of three adjacent landing rays, used in estimating the rate of change of range with takeoff angle. See $f(\rho)$ in notes on subroutine.
K	A control determining the action required when the end of a batch of landing rays is encountered. Possible values are: 1 (mode-split found), 2 (non-landing ray) or 3 (non-existant ray-hop).
LA	DŌ-loop index for takeoff angles.
LH	DŌ-loop index for hops.
LIN	Line counter. When the line count exceeds 43 a fresh page of printout is initiated.
MIB	Number of landing rays in the previous batch.
NB	A control indicating that a collection of landing rays is not (NB = 1) or is (NB = 2) in progress.
NIB	Counter for the number of landing rays in the current batch.
RANGE	Range, in km, of the ray for which the spatial dispersion is being computed.
REX360	$360r_e$ (r_e = 6370 km). Set in DATA statement.
RRE	Reciprocal of earth radius r_e. Set in DATA statement.
SPDB	Array containing the values of spatial dispersion loss, in db, for each ray-hop.
TRIG	Product of the sine of a ray-hop range angle and the sine of its landing angle, divided by the cosine of the ray takeoff angle.

CŌMMŌN/IMŌ/

The following items from the list (qv) are used: DA, F, HEF, IID, JŌBNŌ, LT, NA and NH.

CŌMMŌN/ŌP/

The following items from the list (qv) are used: ABL, ALPHAS, ALPHRL, ALPHRS, HPK, KEY, NDRL, RHŌL and SIGL.

```
      SUBROUTINE TOTSIG
C     ESTIMATES SIGNAL LOSS DUE TO SPATIAL DISPERSION AT END OF EACH RAY
C     HOP AND COMBINES WITH RELEVANT ABSORPTION VALUE TO GIVE TOTAL LOSS
C     VERSION A (REVISION 6/11/69 A.R.C. OF PROGRAM OF FEB 68 R.M.J.,
C                MODIFIED AS SUB SIGSTR 14/2/69 K.B.B.)
C     ESTIMATION OF SPATIAL DISPERSION REQUIRES THREE ADJACENT LANDING
C     RAYS IN SAME MODE
C
      COMMON /IMO/ JOBNO,IID,LT,F,NA,NH,DA,ALPH1,KP,KG,KX,JP,JX,HFF
      COMMON /OP/ KEY(5,100),NDRL(5,100),RHOL(5,100),ALPHAS(100),
     1            PHL(5,100),GPL(5,100),ABL(5,100),HPK(5,100),SL(5,100),
     2            HL(5,100),ALPHDL(5,100),IL(5,100),RHOK(5,100),
     3            ALPHRS(100),ALPHRL(5,100),XPH(5,100),XGP(5,100),
     4            SIGL(5,100)
      DATA REX360(2293200.),RRE(.1569858712715 9E-03)
      DIMENSION SPDB(5,100)
C
C     LOSS DUE TO SPATIAL DISPERSION (IN DBS) IS DERIVED FROM -
C     10*ALOG10((360*RE/DA)*(SIN(RHO/RE)*SIN(ALPHD)/COS(ALPHA))*F(RHOS))
C     RE=6370KM, DA=TAKEOFF ANGLE INCREMENT(DEG), RHO=RANGE OF RELEVANT
C     RAY-HOP, ALPHA=RAY TAKEOFF ANGLE, ALPHD=HOP LANDING ANGLE
C     F(RHOS) IS RANGE FUNCTION WHOSE FORM DEPENDS ON WHETHER LOSS IS
C     CALCULATED FOR FIRST, MIDDLE OR LAST OF THREE ADJACENT RAYS.
C     FOR NON-LANDING RAYS (NDR.GT.1) AND RAYS IN SETS OF LT.3, SIGNAL
C     LOSS IS SET TO DUMMY VALUE OF ZERO
C
C     SET CONSTANT TERM 10*ALOG10(360*RE/DA)
      CONDBS=ALOG10(REX360/DA)*10.
C     CALCULATION OF SIGNAL LOSS
C     START OF HOP LOOP
      DO 25 LH=1,NH
C     INITIALISE BATCH INDICATOR NB=1 (NO DATA), NUMBER IN BATCH NIB=0
      NB=1
      NIB=0
C     START OF ANGLE (RAY) LOOP
      DO 24 LA=1,NA
C     CHECK THAT RAY-HOP EXISTS
      IF(KEY(LH,LA))3,3,1
C     CHECK FOR LANDING RAY
    1 IF(2-NDRL(LH,LA))2,2,6
C     NON-LANDING RAY. SET FLOW CONTROL K=2
    2 K=2
      GO TO 4
C     NON-EXISTANT RAY-HOP. SET FLOW CONTROL K=3
    3 K=3
C     CHECK WHETHER A BATCH OF DATA HAD BEEN STARTED
    4 GO TO (21,5),NB
C     BATCH WAS STARTED. STORE NO. IN BATCH MIB. RESET NB,NIB TO NO DATA
    5 MIB=NIB
      NB=1
      NIB=0
      GO TO 17
```

```
C     LANDING RAY FOUND. INCREMENT NO. IN BATCH, SET INDICATOR NB=2 (ON)
    6 NIB=NIB+1
      NB=2
C     CHECK NO. OF RAYS IN BATCH
      IF(2-NIB)7,7,22
C     TWO OR MORE RAYS IN BATCH. CHECK FOR MODE SPLIT BETWEEN LAST TWO
    7 IF(HPK(LH,LA)-HEF)8,8,9
    8 IF(HPK(LH,LA-1)-HEF)10,10,16
    9 IF(HEF-HPK(LH,LA-1))10,16,16
C     NO MODE SPLIT FOUND. CHECK NO. OF RAYS IN BATCH
   10 IF(NIB-3)14,11,12
C     THREE RAYS IN BATCH. CALCULATE SIGNAL LOSS FOR FIRST
   11 RANGE=RHOL(LH,LA-2)
      FRHO=ABS(RANGE*3.-RHOL(LH,LA-1)*4.+RHOL(LH,LA))
      TRIG=SIN(RANGE*PRE)*SIN(ALPHRL(LH,LA-2))/COS(ALPHRS(LA-2))
      SPDB(LH,LA-2)=ALOG10(TRIG*FRHO)*10.+CONDBS
      SIGL(LH,LA-2)=SPDB(LH,LA-2)+ABL(LH,LA-2)
C     AT LEAST THREE RAYS IN BATCH. FIND SIGNAL LOSS FOR PENULTIMATE ONE
   12 FRHO=ABS(RHOL(LH,LA-2)-RHOL(LH,LA))
      TRIG=SIN(RHOL(LH,LA-1)*PRE)*SIN(ALPHRL(LH,LA-1))/COS(ALPHRS(LA-1))
      SPDB(LH,LA-1)=ALOG10(TRIG*FRHO)*10.+CONDBS
      SIGL(LH,LA-1)=SPDB(LH,LA-1)+ABL(LH,LA-1)
C     CHECK IF CURRENT RAY IS FINAL ONE
      IF(NA-LA)13,13,24
C     FINAL RAY IS IN A BATCH OF AT LEAST THREE. CALCULATE SIGNAL LOSS
   13 RANGE=RHOL(LH,LA)
      FRHO=ABS(RANGE*3.-RHOL(LH,LA-1)*4.+RHOL(LH,LA-2))
      TRIG=SIN(RANGE*PRE)*SIN(ALPHRL(LH,LA))/COS(ALPHRS(LA))
      SPDB(LH,LA)=ALOG10(TRIG*FRHO)*10.+CONDBS
      SIGL(LH,LA)=SPDB(LH,LA)+ABL(LH,LA)
      GO TO 24
C     LESS THAN THREE RAYS IN BATCH. CHECK IF CURRENT RAY IS FINAL ONE
   14 IF(NA-LA)15,15,24
C     FINAL RAY IS IN BATCH OF LESS THAN THREE. SET FLOW CONTROL K=2
   15 K=2
      GO TO 19
C     MODE SPLIT ENCOUNTERED. SET FLOW CONTROL K=1, STORE NO. OF RAYS IN
C     BATCH MIB FOR PREVIOUS MODE AND START COUNT NIP FOR NEW MODE
   16 K=1
      MIB=NIB-1
      NIB=1
C     CHECK NO. OF RAYS IN LAST BATCH
   17 IF(MIB-2)19,18,20
C     ONLY TWO RAYS IN LAST BATCH. SET DUMMY LOSS FOR FIRST
   18 SPDB(LH,LA-2)=0.
      SIGL(LH,LA-2)=0.
C     NOT MORE THAN TWO RAYS IN PREVIOUS BATCH. SET DUMMY LOSS FOR LAST
   19 SPDB(LH,LA-1)=0.
      SIGL(LH,LA-1)=0.
      GO TO 21
C     AT LEAST THREE RAYS IN PREVIOUS BATCH. FIND SIGNAL LOSS FOR LAST
   20 RANGE=RHOL(LH,LA-1)
      FRHO=ABS(RANGE*3.-RHOL(LH,LA-2)*4.+RHOL(LH,LA-3))
      TRIG=SIN(RANGE*PRE)*SIN(ALPHRL(LH,LA-1))/COS(ALPHRS(LA-1))
```

```
      SPDB(LH,LA-1)=ALOG10(TRIG*FRHO)*10.+CONDBS
      SIGL(LH,LA-1)=SPDB(LH,LA-1)+ABL(LH,LA-1)
C     FOLLOW FLOW CONTROL K
   21 GO TO (22,23,24),K
C     ONE RAY IN CURRENT BATCH. CHECK IF FINAL RAY
   22 IF(NA-LA)23,23,24
C     SET DUMMY LOSS FOR CURRENT RAY
   23 SPDB(LH,LA)=0.
      SIGL(LH,LA)=0.
C     NEXT ANGLE (RAY)
   24 CONTINUE
C     NEXT HOP
   25 CONTINUE
C
C     PRINTOUT OF RESULTS
C     START OF HOP LOOP
      DO 32 LH=1,NH
C     START A NEW PAGE FOR EACH HOP AND INITIALISE LINE COUNT
      PRINT 4101,JOBNO,IID,LT,F
      LIN=4
C     START OF ANGLE (RAY) LOOP
      DO 31 LA=1,NA
C     CHECK LINE COUNT
      IF(LIN-43)27,27,26
C     TITLE / NEW PAGE AND RESTART LINE COUNT
   26 PRINT 4101,JOBNO,IID,LT,F
      LIN=4
C     CHECK THAT RAY-HOP EXISTS
   27 IF(KFY(LH,LA))28,28,29
C     FORCE A BLANK LINE FOR NON-EXISTANT RAY-HOP
   28 PRINT 4102
      GO TO 30
C     PRINT DATA FOR THIS RAY-HOP AND ADVANCE LINE COUNT
   29 PRINT 4103,ALPHAS(LA),LH,NDFL(LH,LA),PHOL(LH,LA),ABL(LH,LA),
     1           SPDB(LH,LA),SIGL(LH,LA)
   30 LIN=LIN+1
C     NEXT ANGLE (RAY)
   31 CONTINUE
C     NEXT HOP
   32 CONTINUE
      RETURN
C     PRINTOUT FORMATS
 4101 FORMAT(8H1    JOB,I5,7X16HIONOSPHERE MODEL,I4,5X9HTIME-STEP,I3,
     18X11HFREQUENCY =,F7.3,4H MHZ//6X82HRAY T/O ANGLE  HOP  NDR     PAN
     2GE-KM   ABSORP-DBS   SPAT.DISP-DBS   TOTAL LOSS-DBS/)
 4102 FORMAT(1H )
 4103 FORMAT(1H ,F15.3,I7,I5,F14.4,F12.4,F15.4,F16.4)
      END
```

6.2.6 Subroutine DATAX

This subroutine enables the punching onto cards, for possible use in other programs, of the range, excess phase and group data and total signal strength for each ray-hop, together with appropriate identification data. In addition, printout of the above data is given, this being a combination of the tabulations obtained from subroutines EXPHGP and TOTSIG (qv).

The output may be ordered in one of two ways, either being cycled by hops, ie, hop 1 - ray 1, ray 2, ... ray N; hop 2 - ray 1, ray 2, ... ray N; etc, or by takeoff angles (rays), ie, ray 1 - hop 1, hop 2, ... hop M; ray 2 - hop 1, hop 2, ... hop M; etc.

Header and trailer cards are punched for each set of rays to aid identification of the sections of the output deck which may include output from subroutine RYPNCH (qv).

A system of line counting for page control of the printout is included, and each page produced is titled with suitable identification data.

STORAGE LIST

Local Variables

LA	DŌ-loop index for takeoff angles (ie, rays).
LG	Control to select column heading (see XPGU) for excess group data printout, ie, 1 for (KM) or 2 for (MUSECS).
LH	DŌ-loop index for hops.
LIN	Line counter for page control of printout.
LP	Control to select column heading (see XPGU) for excess phase data printout, ie, 1 for (KM), 2 for (MUSECS) or 3 for (CYCLES).
X	An A8 word consisting of 8 X's, used on header and trailer cards of a batch of output as an aid to identification. Set in DATA statement.
XPGU	Array of A-mode data forming the column headings for the printout of excess phase and group data. Contents are: (KM), (MUSECS) and (CYCLES), set in DATA statement (see LG and LP).

CŌMMŌN/IMŌ/

The list (qv) up to and including JX is used with the exception of ALPH1, DA, JP, KG and KP.

CŌMMŌN/ŌP/

The following items in the list (qv) are used: ALPHAS, KEY, NDRL, RHŌL, SIGL, XGP and XPH.

```
      SUBROUTINE DATAX
C     PRINTS AND PUNCHES FOR EACH RAY-HOP, IDENTIFICATION DATA, RANGE
C     EXCESS PHASE AND GROUP AND TOTAL SIGNAL STRENGTH
C     VERSION A (REVISION 3/11/69 A.R.C. OF SUB XDATA 14/2/69 K.B.B.)
C
      COMMON /IMO/ JOBNO,IID,LT,F,NA,NH,DA,ALPH1,KP,KG,KX,JP,JX
      COMMON /OP/ KEY(5,100),NDRL(5,100),RHOL(5,100),ALPHAS(100),
     1            PHL(5,100),GPL(5,100),ABL(5,100),HPK(5,100),SL(5,100),
     2            HL(5,100),ALPHDL(5,100),IL(5,100),RHOK(5,100),
     3            ALPHRS(100),ALPHRL(5,100),XPH(5,100),XGP(5,100),
     4            SIGL(5,100)
      DATA X(8HXXXXXXXX)
C     COLUMN HEADINGS FOR EXCESS PHASE AND GROUP DATA
      DIMENSION XPGU(3)
      DATA XPGU(1)(8H  (KM)  ),XPGU(2)(8H(MUSECS)),XPGU(3)(8H(CYCLES))
C     SET SUBSCRIPTS FOR REQUIRED HEADINGS
      LP=(KX+1)/2
      LG=((1-KX)/2)*2+KX
C     PUNCH HEADER CARD FOR THIS SET
      PUNCH 4201,F,F,F,F,F,X,X,X,X,X
C     SELECT REQUIRED ORDER OF OUTPUT
      GO TO (1,7),JX
C     ORDER OF DATA - ALL RAY-HOPS FOR EACH HOP IN TURN
C     START OF HOP LOOP
    1 DO 6 LH=1,NH
C     START A NEW PAGE FOR EACH HOP AND INITIALISE LINE COUNT
      PRINT 4101,JOBNO,IID,LT,F,XPGU(LP),XPGU(LG)
      LIN=4
C     START OF ANGLE (RAY) LOOP
      DO 5 LA=1,NA
C     CHECK THAT RAY-HOP EXISTS
      IF(KEY(LH,LA))5,5,2
C     PUNCH DATA FOR THIS RAY-HOP
    2 PUNCH 4202,JOBNO,IID,LT,F,ALPHAS(LA),LH,NDRL(LH,LA),RHOL(LH,LA),
     1            XPH(LH,LA),XGP(LH,LA),SIGL(LH,LA)
C     CHECK LINE COUNT
      IF(LIN-43)4,4,3
C     TITLE A NEW PAGE AND RESTART LINE COUNT
    3 PRINT 4101,JOBNO,IID,LT,F,XPGU(LP),XPGU(LG)
      LIN=4
C     PRINT DATA FOR THIS RAY-HOP AND INCREMENT LINE COUNT
    4 PRINT 4102,ALPHAS(LA),LH,NDRL(LH,LA),RHOL(LH,LA),XPH(LH,LA),
     1            XGP(LH,LA),SIGL(LH,LA)
      LIN=LIN+1
C     NEXT ANGLE (RAY)
    5 CONTINUE
C     NEXT HOP
    6 CONTINUE
      GO TO 14
C     ORDER OF DATA - ALL HOPS FOR EACH ANGLE (RAY) IN TURN
C     INITIALISE LINE COUNT TO FORCE A FRESH PAGE
    7 LIN=36
C     START OF ANGLE (RAY) LOOP
```

```
      DO 13 LA=1,NA
C     CHECK LINE COUNT
      IF(LIN-35)9,9,8
C     TITLE A NEW PAGE AND RESTART LINE COUNT
    8 PRINT 4101,JOBNO,IID,LT,F,XPGU(LP),XPGU(LG)
      LIN=4
C     START OF HOP LOOP
    9 DO 11 LH=1,NH
C     CHECK THAT RAY-HOP EXISTS
      IF(KEY(LH,LA))12,12,10
C     PUNCH AND PRINT DATA FOR THIS HOP AND INCREMENT LINE COUNT
   10 PUNCH 4202,JOBNO,IID,LT,F,ALPHAS(LA),LH,NDRL(LH,LA),RHOL(LH,LA),
     1           XPH(LH,LA),XGP(LH,LA),SIGL(LH,LA)
      PRINT 4102,ALPHAS(LA),LH,NDRL(LH,LA),RHOL(LH,LA),XPH(LH,LA),
     1           XGP(LH,LA),SIGL(LH,LA)
      LIN=LIN+1
C     NEXT HOP
   11 CONTINUE
C     PRINT A BLANK LINE AFTER DATA FOR EACH RAY AND ADVANCE LINE COUNT
   12 PRINT 4103
      LIN=LIN+1
C     NEXT ANGLE (RAY)
   13 CONTINUE
C     PUNCH TRAILER CARD FOR THIS SET
   14 PUNCH 4203,X,X,X,X,X,F,F,F,F,F
      RETURN
C     PRINT FORMATS
 4101 FORMAT(8H1    JOB,I5,7X16HIONOSPHERE MODEL,I4,5X9HTIME-STEP,I3,
     18X11HFREQUENCY =,F7.3,4H MHZ//10X46HTAKEOFF ANGLE  HOP NDR    RANG
     2E-KM    EX PHASE,A8,11H   EX GROUP,A8,19H   SIG STRENGTH-DBS/)
 4102 FORMAT(1H ,F18.3,I8,I4,F14.4,F16.4,F19.4,F18.4)
 4103 FORMAT(1H )
C     PUNCH FORMATS
 4201 FORMAT(5F8.3,5A8)
 4202 FORMAT(2I5,I4,2F8.3,2I3,F12.4,2F11.4,F10.4)
 4203 FORMAT(5A8,5F8.3)
      END
```

6.3 Group (iii) subprograms

6.3.1 IŌD subprogram.

6.3.2 Subroutine PLTRAY.

6.3.3 Subroutine SŌRTDA.

6.3.4 Subroutine PLTITL.

6.3.5 Subroutine PLTDAT.

6.3.6 Subroutine CALFRM

6.3.7 Subroutine PLTFRM.

6.3.1 IŌD subprogram (FIŌD deck)

This subprogram allocates the disk-units required for storing ray-path data (height and range angle of each computed point) for eventual plotting on the SC4060 unit.

All the data to be plotted on one SC4060 frame (range coverage of 1000 km) are stored on one disk, and with 15 disks allocated, up to 15 plotting frames may be obtained (ie, a maximum range coverage of 15000 km).

100 Arcs are made available on each disk, each arc accomodating 512 words, giving a storage capacity of 51200 words per disk. In the extreme case of 100 ray-paths to be plotted, this allows an average of some 250 points per curve per frame, a curve being the section of a ray-path in one frame.

```
0001 B1        IØD,DISK,,,100
0002 B2        IØD,DISK,,,100
0003 B3        IØD,DISK,,,100
0004 B4        IØD,DISK,,,100
0005 B5        IØD,DISK,,,100
0006 B6        IØD,DISK,,,100
0007 B7        IØD,DISK,,,100
0008 B8        IØD,DISK,,,100
0009 B9        IØD,DISK,,,100
0010 B10       IØD,DISK,,,100
0011 B11       IØD,DISK,,,100
0012 B12       IØD,DISK,,,100
0013 B13       IØD,DISK,,,100
0014 B14       IØD,DISK,,,100
0015 B15       IØD,DISK,,,100
0016           END
```

6.3.2 Subroutine PLTRAY

This subroutine, which is called (if plotted output is required) at the end of each ray-hop computed, acts as the control for the set of subroutines used to obtain plots of the ray-paths on the SC4060 plotting unit.

At the first call in a set of ray-path computations (first hop of first ray) the disks used to store the ray data prior to plotting are rewound and frame data, eg, curves per frame, points per curve, are set to zero.

On each call to the subroutine, the height and range angle of each point in the hop are transferred to temporary stores sufficient to accomodate a full ray-path (ie, a maximum of 5 hops with a maximum of 1000 points per hop). On completion of a ray, a call is made to the subroutine SORTDA (qv) which sorts the data and copies it into the disk store.

When a set of ray-paths is complete, calls are made to subroutines PLTITL and PLTDAT (qv), the former giving a title page to the plotted output, the latter plotting the ray data.

STORAGE LIST

Local Variables

LHA Control taking the following values:-

1 - data received is for the first hop of the first ray in a set.

2 - data received is for the first hop of a ray other than the first in a set.

3 - data received is for a hop other than the first of a ray.

Initially set to unity in DATA statement.

MA DO-loop index, used when setting or resetting the array NPTSEC (see COMMON/PLOT/list) to zero.

MFR DO-loop index, used when rewinding disks and setting or resetting the arrays NUMSEC and NPTSEC (see COMMON/PLOT/list) to zero.

J DO-loop index, used when storing the data for a ray-hop.

COMMON/IMO/

The only items in the list (qv) referenced here are NA and NH.

COMMON/HOP/

All items in the list (qv) up to and including THETA are used with the exception of ALPHR.

COMMON/PLOT/

The following items in the list (qv) are defined here: HTS, IPTS, NFR, NPTSEC, NUMSEC and THS.

```
0001        SUBROUTINE PLTRAY
0002 C      CONTROLLING ROUTINE FOR THE PLOTTING OF A SET OF RAY PATHS
0003 C      VERSION A (REVISION 12/11/69 A.R.C. OF SUBS PLOTRA AND CICS
0004 C                 FEB 68 R.M.J.)
0005 C
0006        COMMON /IMO/ JOBNO,IID,LT,F,NA,NH
0007        COMMON /HOP/ ALPHR,NDR,LA,LH,I,H(1000),THETA(1000)
0008        COMMON /PLOT/ NTYP,NTITLE,TITLE(400),MAXH,INCDH,IPTS,HTS(5000),
0009       1              THS(5000),NUMSEC(15),NPTSEC(15,100),NFR
0010        DATA LHA(1)
0011 C
0012 C      CHECK IF FIRST HOP OF FIRST RAY (LHA=1), FIRST HOP OF ANOTHER RAY
0013 C      (LHA=2) OR HOP OTHER THAN THE FIRST OF A RAY (LHA=3)
0014        GO TO (1,4,5),LHA
0015 C      FIRST HOP OF FIRST RAY. SET FRAME COUNT TO ZERO, REWIND DISKS, SET
```

```
C     NUMBERS OF RAY SECTIONS IN FRAMES AND POINTS IN SECTIONS TO ZERO
    1 NFR=0
      DO 3 MFR=1,15
      REWIND MFR
      NUMSEC(MFR)=0
      DO 2 MA=1,NA
      NPTSEC(MFR,MA)=0
    2 CONTINUE
    3 CONTINUE
C     FIRST HOP OF A RAY. SET FIRST POINT DATA AND START RAY POINT COUNT
C     FOR THIS RAY
    4 HTS(1)=0.
      THS(1)=0.
      IPTS=1
C     ANY HOP. STORE DATA FOR EACH POINT, INCREMENT POINT COUNT
C     N.B. EXCEPT FOR FIRST HOP, POINT AT START OF HOP IS IGNORED, BEING
C     IDENTICAL WITH END POINT OF PREVIOUS HOP
    5 DO 6 J=2,I
      IPTS=IPTS+1
      HTS(IPTS)=H(J)
      THS(IPTS)=THETA(J)
    6 CONTINUE
C     CHECK IF END OF RAY, I.E. MAXIMUM HOPS OR NDR.GT.3
      IF(LH-NH)7,9,9
    7 IF(NDR-3)8,8,9
C     RAY INCOMPLETE. RESET INDICATOR FOR NEXT HOP
    8 LHA=3
      RETURN
C     RAY COMPLETE. SORT DATA AND STORE ON DISKS
    9 CALL SORTDA
C     CHECK IF FINAL RAY IN SET
      IF(LA-NA)10,11,11
C     MORE RAYS TO COME. RESET INDICATOR FOR NEXT RAY
   10 LHA=2
      RETURN
C     SET OF RAYS COMPLETE. WRITE A TITLE PAGE FOR PLOTTED OUTPUT AND
C     PLOT THE RAY PATHS. RESET INDICATOR FOR NEXT SET OF RAYS
   11 CALL PLTITL
      CALL PLTDAT
      LHA=1
      RETURN
      END
```

6.3.3 Subroutine SORTDA

This subroutine sorts the coordinate data (heights and range angles) for a complete ray-path into sets. Each set covers a range of 1000 km and is written onto a disk holding the data to be plotted in one frame. Up to 15 disks are made available, giving a maximum range coverage of 15000 km.

After calculating the number of frames (ie, sets) required by the ray, the range angle data are scanned sequentially until two points are found which straddle a frame edge (ie, a range of some multiple of 1000 km). The height of the ray-path at a frame edge is found by interpolation, assuming linearity between the two points, viz,

$$\frac{(h_a + r_e)(h_b + r_e) \sin(\theta_b - \theta_a)}{(h_a + r_e) \sin(\theta_o - \theta_a) + (h_b + r_e) \sin(\theta_b - \theta_o)} - r_e,$$

where θ_o is the range angle to the frame edge; h_a, θ_a and h_b, θ_b are the height and range angle of the point before and the point after the frame edge, and r_e is the earth radius. The interpolated point is then the last in one frame and the first in the next. The scanning process is carried on through each 1000 km bracket necessary until the end of the ray-path is reached.

The number of ray-path sections in each relevant frame is incremented, and the number of points in each section counted.

STORAGE LIST

Local Variables

HFST	Height, in km, of a ray at start of a frame.
HLST	Height, in km, of a ray at end of a frame.
J	Loop index for writing height and range angle data onto disk.
KPT	An indexing parameter for setting first value of index in point sorting loop and last value of index for writing data onto disk.
LPT	Loop indexing parameter, ie, first value of index in point sorting loop.
LSEC	Current number of ray-path sections in a frame.
MFR	DŌ-loop index for frames.
MPT	DŌ-loop index for point sorting.
NFRM	Number of frames covered by current ray.
NPT	Point counter.
RA	Radius, in km, to ray-path point immediately before a frame edge.
RB	Radius, in km, to ray-path point immediately after a frame edge.
THBA	Range angle difference between ray-path points straddling a frame edge.
THBŌ	Difference between the range angle to the ray-path point immediately after a frame edge and the range angle to the frame edge.
THFRM	Range angle coverage of one frame, ie, 1000/6370 radian. Set in DATA statement.
THFST	Range angle of ray at start of frame.
THLST	Range angle of ray at end of frame.
THŌ	Range angle to the end of a frame.
THŌA	Difference between the range angle to a frame edge and the range angle to the ray-path point immediately before the frame edge.

CŌMMŌN/PLŌT/

The following items from the list (qv) are referenced here: HTS, IPTS, NFR, NPTSEC, NUMSEC and THS.

```
      SUBROUTINE SORTDA
C     SORTS THE DATA FOR A COMPLETE RAY INTO BLOCKS, EACH BLOCK COVERING
C     A RANGE OF 1000KM, AND STORES ON APPROPRIATE DISKS. DATA VALUES AT
C     FRAME EDGES, I.E. AT 1000KM POINTS, ARE OBTAINED BY INTERPOLATION
C     VERSION A (REVISION 13/11/69 A.R.C. OF SORTING SECTION OF SUB
C                PLOTDA FEB 68 R.M.J.)
C
      COMMON /PLOT/ NTYP,NTITLE,TITLE(400),MAXH,INCDH,IPTS,HTS(5000),
     1              THS(5000),NUMSEC(15),NPTSEC(15,100),NFR
C
C     RANGE ANGLE COVERED BY ONE FRAME (1000KM)
      DATA THFRM(.15698587127159)
C     DETERMINE NUMBER OF FRAMES COVERED BY THIS RAY
      NFRM=THS(IPTS)*6.37+1.
C     IF NFRM EXCEEDS 15 (MAXIMUM ALLOWED), RESET TO 15
      IF(NFRM-15)2,2,1
    1 NFRM=15
C     UPDATE MAXIMUM NUMBER OF FRAMES REQUIRED IF NECESSARY
    2 NFR=MAX0(NFRM,NFR)
C     INITIALISE DATA AT FRAME EDGE, END OF FRAME RANGE ANGLE AND FIRST
C     VALUE OF POINT LOOP INDEX
      HLST=0.
      THLST=0.
      THO=0.
      KPT=1
C     EXECUTE THE FOLLOWING LOOP FOR EACH FRAME NECESSARY
      DO 6 MFR=1,NFRM
C     UPDATE NUMBER OF RAY SECTIONS IN THIS FRAME
      NUMSEC(MFR)=NUMSEC(MFR)+1
      LSEC=NUMSEC(MFR)
C     SET RANGE ANGLE TO END OF THIS FRAME
      THO=THO+THFRM
C     SET FIRST POINT DATA FOR THIS FRAME AND START POINT COUNT
      HFST=HLST
      THFST=THLST
      NPT=1
C     SET FIRST VALUE OF POINT LOOP INDEX
      LPT=KPT+1
C     FIND THE POINTS TO BE INCLUDED IN THIS FRAME
      DO 5 MPT=LPT,IPTS
C     CHECK IF POINT LIES IN CURRENT FRAME
      THBO=THS(MPT)-THO
      IF(THBO)4,3,3
C     POINT IS IN NEXT FRAME. INTERPOLATE TO FIND DATA AT FRAME EDGE
    3 RA=HTS(MPT-1)+6370.
      RB=HTS(MPT)+6370.
      THOA=THO-THS(MPT-1)
      THBA=THS(MPT)-THS(MPT-1)
      HLST=RA*RB*SIN(THBA)/(RA*SIN(THOA)+RB*SIN(THBO))-6370.
      THLST=THO
C     SET POINT COUNT FOR THIS RAY SECTION, HOLD INDEX OF PREVIOUS POINT
      NPTSEC(MFR,LSEC)=NPT+1
      KPT=MPT-1
C     STORE DATA FOR THIS RAY SECTION ON DISK AND RECYCLE FOR NEXT FRAME
      WRITE(MFR)HFST,THFST,(HTS(J),THS(J),J=LPT,KPT),HLST,THLST
      GO TO 6
C     POINT IS IN THIS FRAME. INCREMENT POINT COUNT AND TRY NEXT POINT
    4 NPT=NPT+1
    5 CONTINUE
C     LAST POINTS FOR THIS RAY. SET POINT COUNT AND STORE DATA ON DISK
      NPTSEC(MFR,LSEC)=NPT
      WRITE(MFR)HFST,THFST,(HTS(J),THS(J),J=LPT,IPTS)
C     NEXT FRAME
    6 CONTINUE
      RETURN
      END
```

6.3.4 Subroutine PLTITL

This subroutine is used to print identification data, ie, job number, ionosphere model number, time-step number, signal frequency and takeoff angle information, and a description, contained in array TITLE (see list for COMMON/PLOT/), on the SC4060 page preceding a set of ray-path plots.

STORAGE LIST

Local Variables

DJOB — Array of A8 words to be typed with identification data of a set of ray-path plots. Defined in DATA statement.

J — DO-loop index for typing DJOB (qv).

KTI — DO-loop index for typing words (A8) in one line of TITLE (see list for COMMON/PLOT/).

LY — Y-coordinate of a line of TITLE.

MTI — DO-loop index for typing lines of TITLE, and loop indexing parameter for typing words in one line, ie, subscript of first word in a line.

NTI — Loop indexing parameter for typing words in one line of TITLE, ie, subscript of last word in a line.

COMMON/IMO/

The list (qv) up to and including ALPH1 is used with the exception variables NA and NH.

COMMON/PLOT/

The first three items in the list (qv) are used.

```
      SUBROUTINE PLTITL
C     PRINTS JOB DESCRIPTION AND TITLE ON FIRST PAGE OF SET OF RAY PLOTS
C     VERSION A (REVISION 13/11/69 A.R.C. OF SUB TITE FEB 68 R.M.J.)
C
      COMMON /IMO/ JOBNO,IID,LT,F,NA,NH,DA,ALPH1
      COMMON /PLOT/ NTYP,NTITLE,TITLE(400)
C
C     JOB DESCRIPTION PRINTOUT
      DIMENSION DJOB(14)
      DATA (DJOB(J),J=1,14)(8HJOB NO. ,8H        ,8H  IID   ,8H       T,
     1          8HIME-STEP,8HFREQUENC,8HY =     ,8H  MHZ   ,8H   T/O A,
     2          8HNGLES FR,8HOM      ,8H DEG IN ,8H     DEG,8H STEPS  )
C     START A FRESH PAGE
      CALL ADVFLM(NTYP)
C     PRINT JOB NO., IONOSPHERE NO. AND TIME-STEP NO. ON FIRST LINE
      CALL TSP(50,48,40)
      DO 1 J=1,5
      CALL HORAM(DJOB(J),8)
    1 CONTINUE
      CALL TSP(106,48,40)
      CALL C4020I(JOBNO,5)
      CALL TSP(218,48,40)
```

```
      CALL C4020I(IID,4)
      CALL TSP(370,48,40)
      CALL C4020I(LT,3)
C     PRINT FREQUENCY AND ANGLE DATA ON SECOND LINE
      CALL TSP(50,48,60)
      DO 2 J=6,14
      CALL HORAM(DJOB(J),8)
    2 CONTINUE
      CALL TSP(138,48,60)
      CALL C4020F(F,7,3)
      CALL TSP(386,48,60)
      CALL C4020F(ALPH1,7,3)
      CALL TSP(490,48,60)
      CALL C4020F(DA,6,3)
C     PRINT CONTENTS OF ARRAY *TITLE*, TEN A8 WORDS PER LINE
      LY=80
      DO 4 MTI=1,NTITLE,10
      NTI=MTI+9
      IF(NTI.GT.NTITLE)NTI=NTITLE
      LY=LY+20
      CALL TSP(50,48,LY)
      DO 3 KTI=MTI,NTI
      CALL HORAM(TITLE(KTI),8)
    3 CONTINUE
    4 CONTINUE
      CALL STPTYP
      RETURN
      END
```

6.3.5 Subroutine PLTDAT

This subroutine enables the plotting of a set of ray-paths in in a series of frames on the SC4060 plotting unit, each frame covering a range of 1000 km.

On the first call to this subroutine in a run, a call is made to subroutine CALFRM (qv) in order to calculate the data for drawing a plotting frame. In subsequent calls to PLTDAT during a run, CALFRM is bypassed, the calculated data being held in COMMON/FRAME/ (qv).

After the drawing of a frame by a call to subroutine PLTFRM (qv), the data for each relevant ray-path section in turn is read from the appropriate disk, converted into SC4060 coordinates and plotted. The process is repeated for the number of frames required by the set of ray-paths. The end of a set of plots is indicated by a blank frame.

STORAGE LIST

Local Variables

MFR	DŌ-loop index for frames.
MPT	Loop index for reading point data off disk and DŌ-loop index for plotting points on ray-path.
MSEC	DŌ-loop index for the ray-path sections in a frame.
NŌFRM	Control for the calculation of the data required for drawing a plotting frame. Set to unity in DATA statement allowing a call to subroutine CALFRM (qv), then reset to 2 for the remainder of the run so that CALFRM is bypassed.
NPTS	Number of points in ray-path section being plotted.
NSEC	Number of ray-path sections to be plotted in the current frame.
RSC	Scaled radius to a point in a ray-path (SC4060 units).
THET	Range angle of a ray-path point relative to the range angle at the start of the frame in which the point is located.
THFRM	Range angle coverage of one frame, ie, 1000/6370 radian. Set in DATA statement.
THŌ	Range angle at the start of a frame.
XA, XB	x-coordinates, in SC4060 units, of two successive points in a ray-path.
YA, YB	y-coordinates, in SC4060 units, of two successive points in a ray-path.

CŌMMŌN/PLŌT/

The following items in the list (qv) are used here: HTS, NFR, NPTSEC, NTYP, NUMSEC and THS.

CŌMMŌN/FRAME/

The first three items in the list (qv) are referenced here.

```
      SUBROUTINE PLTDAT
C     PLOTS RAY PATHS ON A SERIES OF SC4060 FRAMES, EACH FRAME COVERING
C     A RANGE OF 1000KM
C     VERSION A (REVISION 13/11/69 A.R.C. OF DATA PLOTTING SECTION OF
C                SUB PLOTDA FEB 68 R.M.J.)
C
      COMMON /PLOT/ NTYP,NTITLE,TITLE(400),MAXH,INCDH,IPTS,HTS(5000),
     1              THS(5000),NUMSEC(15),NPTSEC(15,100),NFR
      COMMON /FRAME/ LRS,LRF,RHX
C
C     FRAME CALCULATION CONTROL AND RANGE ANGLE EQUIVALENT OF 1000KM
      DATA NOFRM(1),THFRM(.15698587127159)
C     IF THIS IS THE FIRST CALL TO PLTDAT IN THIS RUN, CALCULATE DATA
C     REQUIRED TO DRAW PLOTTING FRAME AND RESET CALFRM CONTROL
      GO TO (1,2),NOFRM
    1 CALL CALFRM
      NOFRM=2
C     INITIALISE ANNOTATION AND ANGLE FOR RANGE AT FRAME START
    2 LRS=-1000
      THO=-THFRM
C     EXECUTE THE FOLLOWING LOOP FOR EACH FRAME REQUIRED
      DO 5 MFR=1,NFR
C     SET RANGE LIMITS (ANNOTATIONS) AND INITIAL RANGE ANGLE FOR THIS
C     FRAME AND DRAW THE FRAME
      LRS=LRS+1000
      LRF=LRS+1000
      THO=THO+THFRM
      CALL PLTFRM
C     REWIND DISK FOR THIS FRAME
      REWIND MFR
C     SET NUMBER OF RAY SECTIONS TO BE PLOTTED IN THIS FRAME
      NSEC=NUMSEC(MFR)
C     EXECUTE THE FOLLOWING LOOP FOR EACH RAY SECTION
      DO 4 MSEC=1,NSEC
C     SET NUMBER OF POINTS IN THIS RAY SECTION
      NPTS=NPTSEC(MFR,MSEC)
C     READ DATA FOR THIS RAY SECTION OFF DISK
      READ(MFR)(HTS(MPT),THS(MPT),MPT=1,NPTS)
C     FIND X,Y COORDINATES OF FIRST POINT
      RSC=HTS(1)*RHX+5096.
      THET=THS(1)-THO
      XB=SIN(THET)*RSC+60.
      YB=5946.-COS(THET)*RSC
C     FIND X,Y COORDINATES OF SUCCESSIVE POINTS AND PLOT RAY SECTION
      DO 3 MPT=2,NPTS
      XA=XB
      YA=YB
      RSC=HTS(MPT)*RHX+5096.
      THET=THS(MPT)-THO
      XB=SIN(THET)*RSC+60.
      YB=5946.-COS(THET)*RSC
      CALL VECTOR(XA,YA,XB,YB)
    3 CONTINUE
C     NEXT RAY SECTION
    4 CONTINUE
C     NEXT FRAME
    5 CONTINUE
C     THROW A BLANK PAGE AFTER EACH SET OF RAY PLOTS
      CALL ADVFLM(NTYP)
      RETURN
      END
```

6.3.6 Subroutine CALFRM

This subroutine calculates all the data necessary for drawing plotting frames for the ray-paths on the SC4060 plotter unit.

One frame covers a fixed range of 1000 km, scaled to 800 plotter units. The height coverage required is input, and is also scaled to 800 plotter units. Therefore, for true height-range plots a maximum heights of 1000 km should be specified. Usually, however, an exaggerated height scale is used.

The frame origin (zero height, zero range) is set at X = 60, Y = 850 on a plotting page, so that the earth centre is located (off frame) at X = 60, Y = 5946 (since 6370 km ≡ 5096 plotter units). Thus, any point height h, range angle θ (from frame start), has coordinates on the plotting page of

$$X = 60 + \left(5096 + \frac{800}{h_{MAX}} \cdot h\right) \sin\theta$$

$$Y = 5946 - \left(5096 + \frac{800}{h_{MAX}} \cdot h\right) \cos\theta,$$

where h_{MAX} is the maximum height of the frame.

Height-scale marks, at specified intervals, are set at the left, on either side of the centre and at the right of the frame. The interval between scale marks must be such that not more than 9 marks are required between h = 0 and h = h_{MAX}, ie, not more than 10 divisions in the scale. The scale marks have a surface length of 12.5 km, ie, 10 plotter units, and are annotated at the left side of the frame.

Range scales, consisting of segments of surface length 10 km, are drawn at h = 0 (ie, surface) and at h = h_{MAX}. Scale marks are added at 100 km intervals and are of length 15 plotter units at the ends, 10 units in the centre and 5 units elsewhere. Annotations are put at the ends of the surface scale.

The data calculated in CALFRM are the height scale factor, the number and values of the height-scale marks, the coordinates of the end points of the height and range-scale marks, the coordinates of the points forming the range scales, and the coordinates for annotating the right hand end of the surface range scale.

STORAGE LIST

Local Variables

ANG	Range angle, in radians, from the start of a frame to a point forming part of a range scale.
CAN	cosine of ANG (qv).
CHCW	Product of cosine of RAH (qv) and cosine of W (qv).
CHSW	Product of cosine of RAH (qv) and sine of W (qv).
CRA	cosine of RA (qv).
CRAH	cosine of RAH (qv).
CRAHMW	cosine of RAH - W.
CRAHPW	cosine of RAH + W.
CRAMW	cosine of RA - W.
CW	cosine of W (qv).
DAN	Angular width, in radians, of segments forming range scales (10/6370 radian). Set in DATA statement.
HMAX	Maximum height, in km, of height scale.
HSC	Height, in plotter units, of a height-scale mark.
IA	Loop indexing parameter for range scale points, ie, subscript of first in a set of 10 forming range scale between 2 scale marks.
IB	Loop indexing parameter for range-scale points, ie, subscript of last in a set of 10 forming range scale between 2 scale marks.
MHS	DŌ-loop index for height-scale marks.
MRS	DŌ-loop index for range-scale points.
MS	DŌ-loop index for range-scale marks.
RA	Range angle, in radians, covered by one frame (1000/6370 radian). Set in DATA statement.
RAH	Half-frame range angle, in radians (500/6370 radian). Set in DATA statement.
RSC	Radius, in plotter units, from earth centre to a height-scale mark.

RSCM — Array containing radii, in plotter units, to upper ends of upper range-scale marks. Set in DATA statement.

RSCO — Array containing radii, in plotter units, to lower ends of surface range-scale marks. Set in DATA statement.

SAN — sine of ANG (qv).

SHCW — Product of sine of RAH (qv), and cosine of W (qv).

SHSW — Product of sine of RAH (qv), and sine of W (qv).

SRA — sine of RA (qv).

SRAH — sine of RAH (qv).

SRAHMW — sine of RAH - W

SRAHPW — sine of RAH + W.

SRAMW — sine of RA - W.

SW — sine of W (qv).

W — Angular width, in radians, of height-scale marks (12.5/6370 radian). Set in DATA statement.

COMMON/PLOT/

Only MAXH and INCDH are used.

COMMON/FRAME/

The full list is used with the exception of LRF and LRS.

```
      SUBROUTINE CALFRM
C     COMPUTES SC4060 COORDINATES OF ALL POINTS REQUIRED FOR DRAWING A
C     PLOTTING FRAME FOR THE RAY PATHS. A FRAME COVERS A FIXED RANGE OF
C     1000KM SCALED TO 800 SC4060 UNITS. THE HEIGHT SCALE IS VARIABLE,
C     THE MAXIMUM HEIGHT REQUIRED BEING SCALED TO 800 SC4060 UNITS
C     (I.E. FOR A TRUE HEIGHT VS RANGE PLOT, A MAXIMUM HEIGHT OF 1000KM
C     SHOULD BE SPECIFIED)
C     VERSION A (REVISION 13/11/69 A.R.C. OF PARTS OF SUB PLOTDA FEB 68
C                R.M.J.)
C
      COMMON /PLOT/ NTYP,NTITLE,TITLE(400),MAXH,INCDH
      COMMON /FRAME/ LRS,LRF,RHX,NHS,LHS(9),YHL(9),XLR(9),YLR(9),XCL(9),
     1               YCL(9),XHC(9),YHC(9),XCR(9),YCR(9),XRL(9),YRL(9),
     2               XHR(9),YHR(9),XRO(101),YRO(101),XRM(101),YRM(101),
     3               XOA(10),YOA(10),XOB(10),YOB(10),XMA(10),YMA(10),
     4               XMB(10),YMB(10),XRX,YRX
C
C     ORIGIN OF FRAME, I.E. H=0, RANGE=0, IS SET AT X=60, Y=850, SO THAT
```

```
C     EARTH CENTRE IS LOCATED AT (60,5946) (6370KM=5096 PLOTTER UNITS)
C
C     RANGE SCALE MARKS ARE SET AT 100KM INTERVALS AND HAVE LENGTHS (IN
C     PLOTTER UNITS) OF 5 (MINOR), 10 (HALF RANGE) AND 15 (FULL RANGE)
C     RADII TO ENDS OF THESE MARKS ARE IN RSCM (UPPER RANGE SCALE) AND
C     IN RSCO (SURFACE RANGE SCALE)
      DIMENSION RSCM(10),RSCO(10)
      DATA (RSCM(J),J=1,10)(/4/5901.,5906.,/4/5901.,5911.),
     1     (RSCO(J),J=1,10)(/4/5091.,5086.,/4/5091.,5081.)
C     RANGE AXES ARE FORMED OF SEGMENTS WITH A SURFACE LENGTH OF 10KM,
C     ANGULAR WIDTH DAN=10/6370 RADS
      DATA DAN(.15698587127159E-02)
C     HEIGHT SCALE MARKS HAVE LENGTH 10 PLOTTER UNITS (ANGULAR WIDTH
C     W=10/5096 RADS) AND ARE DRAWN AT FRAME LEFT (FROM ANGLE 0 TO W),
C     ON EITHER SIDE OF FRAME CENTRE (FROM RAH-W TO RAH TO RAH+W, RAH IS
C     HALF RANGE ANGLE) AND AT FRAME RIGHT (FROM RA-W TO RA, RA IS FULL
C     RANGE ANGLE)
      DATA W(.00196232339089),RAH(.07849293563579),RA(.15698587127159)
C
C     SET MAXIMUM HEIGHT (HMAX), NUMBER OF HEIGHT SCALE MARKS (NHS) AND
C     HEIGHT SCALE (RHX, PLOTTER UNITS/KM)
      HMAX=MAXH
      NHS=MAXH/INCDH-1
      RHX=800./HMAX
C     CALCULATE SINES AND COSINES OF ANGLES REQUIRED FOR HEIGHT SCALES
      SW=SIN(W)
      CW=COS(W)
      SRAH=SIN(RAH)
      CRAH=COS(RAH)
      SRA=SIN(RA)
      CRA=COS(RA)
      CHCW=CRAH*CW
      SHSW=SRAH*SW
      SHCW=SRAH*CW
      CHSW=CRAH*SW
      CRAHMW=CHCW+SHSW
      SRAHMW=SHCW-CHSW
      CRAHPW=CHCW-SHSW
      SRAHPW=SHCW+CHSW
      CRAMW=CRA*CW+SRA*SW
      SRAMW=SRA*CW-CRA*SW
C     CALCULATE COORDINATES FOR HEIGHT SCALE DATA
      DO 1 MHS=1,NHS
C     HEIGHT VALUE (LHS), SCALED VALUE (HSC), SCALED RADIUS (RSC) AND Y
C     COORDINATE ON HEIGHT AXIS (YHL)
      LHS(MHS)=MHS*INCDH
      HSC=FLOAT(LHS(MHS))*RHX
      RSC=HSC+5096.
      YHL(MHS)=850.-HSC
C     LEFT SIDE SCALE MARKS
      XLR(MHS)=SW*RSC+60.
      YLR(MHS)=5946.-CW*RSC
C     CENTRE SCALE MARKS
      XCL(MHS)=SRAHMW*RSC+60.
```

```
      YCL(MHS)=5946.-CRAHMW*RSC
      XHC(MHS)=SRAH*RSC+60.
      YHC(MHS)=5946.-CRAH*RSC
      XCR(MHS)=SRAHPW*RSC+60.
      YCR(MHS)=5946.-CRAHPW*RSC
C     RIGHT SIDE SCALE MARKS
      XRL(MHS)=SRAMW*RSC+60.
      YRL(MHS)=5946.-CRAMW*RSC
      XHR(MHS)=SRA*RSC+60.
      YHR(MHS)=5946.-CRA*RSC
    1 CONTINUE
C     CALCULATE COORDS OF POINTS FORMING RANGE SCALES AND SCALE MARKS
C     INITIALISE RANGE SCALE LOOP INDEX AND RANGE ANGLE
      IA=-8
      ANG=0.
C     SET FIRST POINTS OF SCALES
      XRO(1)=60.
      YRO(1)=850.
      XRM(1)=60.
      YRM(1)=50.
C     LOOP FOR RANGE SCALE MARKS (100KM INTERVALS)
      DO 3 MS=1,10
C     SET LOOP INDICES FOR SCALE POINTS TO NEXT SCALE MARK
      IA=IA+10
      IB=IA+9
C     LOOP FOR RANGE SCALE POINTS (10KM INTERVALS)
      DO 2 MRS=IA,IB
C     SET RANGE ANGLE FOR THIS POINT, FIND SINE AND COSINE
      ANG=ANG+DAN
      SAN=SIN(ANG)
      CAN=COS(ANG)
C     LOWER (SURFACE) SCALE POINT COORDINATES
      XRO(MRS)=5096.*SAN+60.
      YRO(MRS)=5946.-5096.*CAN
C     UPPER (MAXIMUM HEIGHT) SCALE POINT COORDINATES
      XRM(MRS)=5896.*SAN+60.
      YRM(MRS)=5946.-5896.*CAN
    2 CONTINUE
C     COORDINATES OF SURFACE SCALE MARKS
      XOA(MS)=XRO(IB)
      YOA(MS)=YRO(IB)
      XOB(MS)=RSCO(MS)*SAN+60.
      YOB(MS)=5946.-RSCO(MS)*CAN
C     COORDINATES OF UPPER RANGE SCALE MARKS
      XMA(MS)=XRM(IB)
      YMA(MS)=YRM(IB)
      XMB(MS)=RSCM(MS)*SAN+60.
      YMB(MS)=5946.-RSCM(MS)*CAN
    3 CONTINUE
C     COORDINATES FOR ANNOTATION OF RIGHT HAND END OF SURFACE SCALE
      XRX=XOB(10)-56.
      YRX=YOB(10)
      RETURN
      END
```

6.3.7 Subroutine PLTFRM

This subroutine uses the data computed in subroutine CALFRM (qv) to draw a plotting frame for each batch of ray-path data contained in a 1000 km range bracket.

A height scale is drawn at the left of the plotting page, height marks are added to this scale and are also drawn at half-range and full range, and the height scale is annotated. A range scale, approximating an arc of a circle and consisting of short linear segments, is drawn to represent the earth surface, and a further scale is drawn at the frame maximum height. These scales have marks added at the 100 km points and the extreme marks on the surface scale are annotated with range values appropriate to the frame.

Data scaling and frame positioning on the SC4060 raster are described in the notes on subroutine CALFRM.

STORAGE LIST

Local Variables

MHS DŌ-loop index for height-scale marks.

MRS DŌ-loop index for drawing segments of range scales.

MS DŌ-loop index for range-scale marks.

CŌMMŌN/PLŌT/

The only items in the list (qv) which are referenced here are MAXH and NTYP.

CŌMMŌN/FRAME/

The full list (qv) is used with the exception of RHX.

```
      SUBROUTINE PLTFRM
C     DRAWS A PLOTTING FRAME COVERING A RANGE OF 1000KM
C     VERSION A (REVISION 13/11/69 A.R.C. OF FRAME PLOTTING SECTIONS OF
C                SUB PLOTDA FEB 68 R.M.J.)
C
      COMMON /PLOT/ NTYP,NTITLE,TITLE(400),MAXH
      COMMON /FRAME/ LRS,LRF,RHX,NHS,LHS(9),YHL(9),XLR(9),YLR(9),XCL(9),
     1               YCL(9),XHC(9),YHC(9),XCR(9),YCR(9),XRL(9),YRL(9),
     2               XHR(9),YHR(9),XRO(101),YRO(101),XRM(101),YRM(101),
     3               XOA(10),YOA(10),XOB(10),YOB(10),XMA(10),YMA(10),
     4               XMB(10),YMB(10),XRX,YRX
C
C     THROW A FRESH PAGE AND CONFIRM PLOT MODE
      CALL ADVFLM(NTYP)
      CALL STPTYP
C     DRAW HEIGHT AXIS
      CALL VECTOR(60.,35.,60.,865.)
C     DRAW HEIGHT SCALE MARKS AT LEFT, CENTRE AND RIGHT OF FRAME
      DO 1 MHS=1,NHS
C     LEFT SIDE HEIGHT SCALE MARKS
      CALL VECTOR(XLR(MHS),YLR(MHS),60.,YHL(MHS))
C     CENTRE HEIGHT SCALE MARKS
      CALL VECTOR(XCL(MHS),YCL(MHS),XHC(MHS),YHC(MHS))
      CALL VECTOR(XCR(MHS),YCR(MHS),XHC(MHS),YHC(MHS))
C     RIGHT SIDE HEIGHT SCALE MARKS
      CALL VECTOR(XRL(MHS),YRL(MHS),XHR(MHS),YHR(MHS))
    1 CONTINUE
C     LABEL HEIGHT SCALE
C     ZERO MARK
      CALL TSP(45.,0,850.)
C     INTERMEDIATE MARKS
      DO 2 MHS=1,NHS
      CALL TSP(5.,48,YHL(MHS))
      CALL C4020I(LHS(MHS),5)
```

```
    2 CONTINUE
C     MAXIMUM HEIGHT MARK
      CALL TSP(5.,48,50.)
      CALL C4020I(MAXH,5)
C     RESTORE PLOTTING MODE
      CALL STPTYP
C     DRAW RANGE SCALES AT SURFACE AND MAXIMUM HEIGHT
      DO 3 MRS=2,100,2
C     SURFACE SCALE
      CALL VECTOR(XRO(MRS-1),YRO(MRS-1),XRO(MRS),YRO(MRS))
      CALL VECTOR(XRO(MRS+1),YRO(MRS+1),XRO(MRS),YRO(MRS))
C     SCALE AT MAXIMUM HEIGHT
      CALL VECTOR(XRM(MRS-1),YRM(MRS-1),XRM(MRS),YRM(MRS))
      CALL VECTOR(XRM(MRS+1),YRM(MRS+1),XRM(MRS),YRM(MRS))
    3 CONTINUE
C     DRAW RANGE SCALE MARKS
      DO 4 MS=1,10
      CALL VECTOR(XOB(MS),YOB(MS),XOA(MS),YOA(MS))
      CALL VECTOR(XMB(MS),YMB(MS),XMA(MS),YMA(MS))
    4 CONTINUE
C     ANNOTATE START AND FINISHING MARKS ON SURFACE RANGE SCALE
      CALL TSP(4.,48,865.)
      CALL C4020I(LRS,6)
      CALL TSP(XRX,48,YRX)
      CALL C4020I(LRF,6)
C     RESTORE PLOTTING MODE
      CALL STPTYP
      RETURN
      END
```

6.4 Group (iv) subprograms

This group of subprograms contains examples of two types of ionospheric model:-

(a) A three-layer model containing a twilight region.

(b) As (a) but having in addition a model of a sporadic E-layer.

Subprograms applicable to both models:-

6.4.1 Subroutine TVP (version B).

6.4.2 Function HFTH.

6.4.3 Subroutine FOFHTH (version B).

6.4.4 Subroutine EMH (version B).

Subroutines applicable to example (a):-

6.4.5 Subroutine SETC (version DEF).

6.4.6 Subroutine NRX (version NF).

6.4.7 Subroutine NHBP (version DEF/L).

Subroutines applicable to example (b):-

6.4.8 Subroutine SETC (version DEF/ES).

6.4.9 Subroutine NRX (version NF + ES).

6.4.10 Subroutine NHBP (version DEF/H).

6.4.11 Subroutine NHES.

6.4.1 Subroutine TVP (version B)

Subroutine TVP provides for the systematic variation of a number of parameters of the ionospheric model used.

This version is for use with models containing a twilight transition region, and allows this region to be shifted by equal increments of range for equal increments in time-step number.

STORAGE LIST

Local Variables

KT — Previous time-step number, initially set to unity in DATA statement.

RANGE — Range, in km, to centre of twilight transition region at current time-step.

RE — Earth radius, 6370 km, set in DATA statement.

SHIFT — Angular change in position of twilight transition region from previous position.

Blank COMMON

C — Elements 401 to 404 of this array are used. These are named locally as:-

- C401 — Range angle to centre of twilight transition region.
- C402 — Range angle to near boundary of transition.
- C403 — Range angle to far boundary of transition.
- C404 — Angular shift per time-step of transition region.

COMMON/IMO/

The only items in the list (qv) that are referenced are IID and LT.

```
      SUBROUTINE TVP
C     CALCULATES CURRENT VALUES OF IONOSPHERIC PARAMETERS THAT VARY WITH
C     TIME-STEP LT
C     VERSION B (REVISION 4/11/69 A.R.C. OF SUB TDPS (B) 1/12/67 A.R.C.)
C     POSITION OF TWILIGHT TRANSITION IS TIME VARIABLE
C
      COMMON C(400),C401,C402,C403,C404
      COMMON /IMO/ JOBNO,IID,LT
      DATA KT(1),RE(6370.)
C
C     RANGE ANGLE TO TRANSITION CENTRE IS IN C(401), LOWER AND UPPER
C     LIMITS IN C(402) AND C(403) AND SHIFT PER TIME-STEP IN C(404)
      SHIFT=FLOAT(LT-KT)*C404
      C401=SHIFT+C401
      RANGE=C401*RE
      C402=C402+SHIFT
      C403=C403+SHIFT
C     RESET KT AND PRINT NEW POSITION
      KT=LT
      PRINT 4101,IID,LT,RANGE
      RETURN
 4101 FORMAT(1H1/////30H  ******* FOR IONOSPHERE MODEL,I4,13H AT TIME-ST
     1EP,I3,31H, RANGE TO TRANSITION CENTRE IS,F8.1,11H KM *******/////)
      END
```

6.4.2 Function HFTH

This function determines the height of the ionospheric boundary labelled L, at a given range angle θ.

In this version, used with ionospheric models whose boundaries are concentric with the earth, the height of any given boundary is constant, so the value of θ is immaterial. The heights of the boundaries, in ascending order, are stored in consecutive cells of the blank COMMON array C, starting at C(101).

STORAGE LIST

Argument Cells

HFTH	(output)	Height, in km, of ionospheric boundary L at range angle THETA.
L	(input)	Label of ionospheric boundary.
THETA	(input)	Current value of range angle, in radians.

Blank COMMON

C — Consecutive elements, from C(101) are required in this version, so the array is divided into two parts, the second of which commences with the required elements and is given the local name C101.

```
      FUNCTION HFTH(L,THETA)
C     FINDS HEIGHT OF IONOSPHERIC BOUNDARY *L* AT RANGE ANGLE *THETA*
C     VERSION A2 (REVISED 21/10/69 A.R.C.)
C     BOUNDARIES CONCENTRIC WITH EARTH. HEIGHTS STORED IN C(101) FT. SEC
      COMMON C(100),C101(900)
      HFTH=C101(L)
      RETURN
      END
```

6.4.3 Subroutine FŌFHTH (version B)

This subroutine calculates the value and derivatives of the function, variable in height h and range angle θ, used to obtain the electron density in a twilight transition region from the basic (day-time) ionospheric profile.

In this version, the function is a cubic in range angle θ between the day and night regions, the value in the day region being unity, and in the night region, a function of height m(h), contained in subroutine EMH (qv), ie,

in day region:

$$F(h,\theta) = 1$$

$$\frac{\partial F(h,\theta)}{\partial h} = 0$$

$$\frac{\partial F(h,\theta)}{\partial \theta} = 0,$$

In night region:

$$F(h,\theta) = m(h)$$

$$\frac{\partial F(h,\theta)}{\partial h} = \frac{dm(h)}{dh}$$

$$\frac{\partial F(h,\theta)}{\partial \theta} = 0,$$

in transition:

$$F(h,\theta) = \frac{1 + m(h)}{2} \pm [1 - m(h)](\theta_o - \theta)\left[\frac{3}{4\Delta\theta} - \frac{(\theta_o - \theta)^2}{4(\Delta\theta)^3}\right]$$

$$\frac{\partial F(h,\theta)}{\theta h} = \left\{\frac{1}{2} \mp (\theta_o - \theta)\left[\frac{3}{4\Delta\theta} - \frac{(\theta_o - \theta)^2}{4(\Delta\theta)^3}\right]\right\}\frac{dm(h)}{dh}$$

$$\frac{\partial F(h,\theta)}{\partial \theta} = \mp [1 - m(h)]\left[\frac{3}{4\Delta\theta} - \frac{3(\theta_o - \theta)^2}{4(\Delta\theta)^3}\right],$$

where θ_o is the range angle to the transition centre, located in C(401), $\Delta\theta$ is the angular half width of the transition region and the lower and upper limits of the region, $\theta_o - \Delta\theta$ and $\theta_o + \Delta\theta$, are located in C(402) and C(403) respectively. The constants $3/4\Delta\theta$ and $1/4(\Delta\theta)^3$ are located in C(405) and C(406) respectively. The upper sign is used for a day-to-night transition and the lower sign for a night-to-day transition.

STORAGE LIST

Local Variables

A1, A2, A3, A4, A5 — Intermediate variables used in transition region.

TH — Difference between range angle to centre of transition region and current value of range angle.

Blank COMMON

C — Elements 401, 402, 403, 405 and 406 of this array are used and are referenced by the local names C401, C402, etc. Their definitions are given in the notes for the subroutine.

IC — Element 3 of this array is used and is referenced here by the equivalent name ND. It is a control indicating the type of transition required, having the value 1 for day-to-night transition or 2 for night-to-day transition.

COMMON/PT/

The items in the list (qv) that are referenced here are DFDH, DFDTH, DMDH, EM, FHTH and THETA.

```
      SUBROUTINE FOFHTH
C     FUNCTION OF HEIGHT (H) AND RANGE ANGLE (THETA) USED TO OBTAIN THE
C     ELECTRON DENSITY AND ITS DERIVATIVES IN TRANSITION OR NIGHT-TIME
C     REGIONS FROM A BASIC DAY-TIME IONOSPHERIC MODEL
C     VERSION B (REVISION 4/11/69 A.R.C. OF SUB FHTHFT (F) 1/12/67)
C     F(H,THETA) VARIES FROM UNITY IN DAY REGION TO M(H) IN NIGHT REGION
C     AS  F(H,THETA) = (1+M(H))/2 + OR - (1-M(H))*(C(401)-THETA)*
C                                        (C(405)-C(406)*(C(401)-THETA)**2)
C     (+ DAY-TO-NIGHT, - NIGHT-TO-DAY) FOR C(402).LT.THETA.LT.C(403)
C
      COMMON C(400),C401,C402,C403,C404,C405,C406,C407(594),IC(15)
      COMMON /PT/ H,THETA,EN,EMU,RMU,DMUDH,DMUDTH,ENH,DNDH,FHTH,DFDH,
     1            DFDTH,EM,DMDH
      EQUIVALENCE (IC(3),ND)
C
C     LOCATE THETA RELATIVE TO TRANSITION
      IF(THETA-C402)1,1,2
    1 GO TO (4,5),ND
    2 IF(THETA-C403)6,3,3
    3 GO TO (5,4),ND
C     DAY REGION. F(H,THETA) IS UNITY
    4 FHTH=1.
      DFDH=0.
      DFDTH=0.
      RETURN
C     NIGHT REGION. F(H,THETA)=M(H)
    5 CALL EMH
```

```
      FHTH=EM
      DFDH=DMDH
      DFDTH=0.
      RETURN
C     TRANSITION REGION
    6 TH=C401-THETA
      CALL EMH
      A1=1.-EM
      A2=C406*TH*TH
      A3=-(A2-C405)
      A4=A3*TH
      A5=A4*A1
      GO TO (7,8),ND
C     DAY-TO-NIGHT TRANSITION
    7 FHTH=(1.+EM)*.5+A5
      DFDH=(.5-A4)*DMDH
      DFDTH=(A2+A2-A3)*A1
      RETURN
C     NIGHT-TO-DAY TRANSITION
    8 FHTH=(1.+EM)*.5-A5
      DFDH=(.5+A4)*DMDH
      DFDTH=(A3-A2-A2)*A1
      RETURN
      END
```

6.4.4 Subroutine EMH (version B)

This subroutine contains the function m(h) defining the ratio of night-to-day electron densities.

In this version, the function varies linearly with height h, from a value m_o at the ionospheric base height h_o, to a value m_F at the height h_F of the day-time F-layer peak, ie,

$$m(h) = \frac{m_F - m_o}{h_F - h_o}.h + \frac{m_o h_F - m_F h_o}{h_F - h_o}$$

and $$\frac{dm(h)}{dh} = \frac{m_F - m_o}{h_F - h_o}.$$

The constants $\frac{m_F - m_o}{h_F - h_o}$ and $\frac{m_o h_F - m_F h_o}{h_F - h_o}$ are located in C(301) and C(302) respectively.

STORAGE LIST

Blank COMMON

C Elements 301 and 302 of this array are used and are locally named C301 and C302, being, respectively, the slope and constant term of the linear function defining the ratio of night-to-day electron densities.

COMMON/PT/

The only items in the list (qv) referenced here are DMDH, EM and H.

```
      SUBROUTINE EMH
C     CALCULATES RATIO OF NIGHT TO DAY ELECTRON DENSITIES M(H) AND ITS
C     DERIVATIVE AT HEIGHT H
C     VERSION B (REVISION 5/11/69 A.R.C. OF SUB MH (B) 1/12/67 A.R.C.)
C     RATIO VARIES LINEARLY WITH HEIGHT FROM MO AT IONOSPHERE BASE TO MF
C     AT HEIGHT OF F-LAYER MAXIMUM AS  M(H) = C(301)*H+C(302)
C
      COMMON C(300),C301,C302
      COMMON /PT/ H,THETA,EN,EMU,RMU,DMUDH,DMUDTH,ENH,DNDH,FHTH,DFDH,
     1            DFDTH,EM,DMDH
C
C     CALCULATE RATIO (EM) AND DERIVATIVE (DMDH)
      DMDH=C301
      EM=DMDH*H+C302
      RETURN
      END
```

6.4.5 Subroutine SETC (version DEF)

This subroutine calculates various constants from the input ionospheric model data and stores them in the blank COMMON array C (qv) for later use. This version is for an ionospheric model based on a 3-layer day-time profile consisting of a D-layer (quadratic function of height, h), E and F-layers (both cubic functions of height). A night-time model is produced by multiplying the day-time profile by a linear function of height, m(h). In the twilight transition region between day and night, the electron density is obtained by multiplying the day-time value by a function $f(h,\theta)$ which, at any height h, is a cubic function of range angle θ varying from unity (day) to m(h) (night). The ionospheric boundaries, ie, base, D to E and E to F divisions and F maximum, are concentric with the earth. The program is in four sections as follows:-

(a) The boundary heights, required for function HFTH (qv), are stored in consecutive elements of array C commencing at C(101). This is a straight transfer of input data, ie,

base height h_o, input in C(4), set in C(101).
D to E boundary h_D, input in C(5), set in C(102).
E to F boundary h_E, input in C(7), set in C(103).
F-layer maximum h_F, input in C(9), set in C(104).

Note: This particular model is not intended for ray-tracing above the height of the F-layer maximum.

(b) The constants for the day-time profile are stored in cells C(201) to C(205), viz,

(i) For the D-layer, where N_D, input in C(6), is the electron density at the top of the layer,

$$\text{electron density} = \frac{N_D\,(h - h_o)^2}{(h_D - h_o)^2}.$$

The constant $\dfrac{N_D}{(h_D - h_o)^2}$ is set in C(201).

(ii) For the E-layer, where N_E, input in C(8), is the electron density at the top (maximum) of the layer,

$$\text{electron density} = N_E - (h_E - h)^2\,[a(h_E - h) + b].$$

The constants

$$a = \frac{2}{(h_E - h_D)^2}\left[\frac{N_D}{h_D - h_o} - \frac{N_E - N_D}{h_E - h_D}\right]$$

and

$$b = \frac{1}{h_E - h_D}\left[\frac{3(N_E - N_D)}{h_E - h_D} - \frac{2N_D}{h_D - h_o}\right]$$

are set in C(202) and C(203) respectively.

(iii) For the F-layer, where N_F, input in C(10), is the electron density at the maximum of the layer,

$$\text{electron density} = N_F - (h_F - h)^2 \, [b - a(h_F - h)].$$

The constants

$$a = \frac{2(N_F - N_E)}{(h_F - h_E)^3} \quad \text{and} \quad b = \frac{3(N_F - N_E)}{(h_F - h_E)^2}$$

and set in C(204) and C(205) respectively.

(c) The factor m(h) to obtain the night-time electron density from the day-time value, varies linearly with height from m_o, input in C(11), at the ionosphere base to m_F, input in C(12), at the F-layer day-time maximum, ie,

$$m(h) = \left(\frac{m_F - m_o}{h_F - h_o}\right) h + \frac{h_F m_o - h_o m_F}{h_F - h_o}.$$

The constants $\frac{m_F - m_o}{h_F - h_o}$ and $\frac{h_F m_o - h_o m_F}{h_F - h_o}$ are set in C(301) and C(302) respectively.

(d) In the transition region, the electron density at height h and range angle θ is obtained by multiplying the day-time value (at h) by

$$f(h,\theta) = \frac{1 + m(h)}{2} \pm [1 - m(h)](\theta_o - \theta) \left[\frac{3}{4\Delta\theta} - \frac{(\theta_o - \theta)^2}{4(\Delta\theta)^3}\right]$$

(+ sign for a day-to-night transition, − sign for night-to-day). θ_o, the range angle to the transition centre, is set in C(401) and is obtained from the corresponding range ρ_o, input in C(13). Δθ is the angular half-width of the transition region and is obtained from the corresponding range half-width Δρ, input in C(14).

The lower $(\theta_o - \Delta\theta)$ and upper $(\theta_o + \Delta\theta)$ angular limits of the transition region are set in C(402) and C(403) respectively, and the constants $3/4\Delta\theta$ and $1/4(\Delta\theta)^3$ are set in C(405) and C(406) respectively. The angular shift of the transition region per time-step is set in C(404) and is obtained from the corresponding range shift $\Delta\rho_T$, input in C(15).

The forms of the vertical profiles (day and night) are shown in figure 8, where the data of example 1, section 7 are used.

STORAGE LIST

Local Variables

A1, A2, A3, A4 — Intermediate variables of various definitions.

RRE — Reciprocal of earth radius (6370 km), set in DATA statement.

Blank COMMON

C — This is the only item in the list (qv) which is used here. The array is segmented so that elements may be referenced without subscripting. The elements defined in this program are: C(101) to C(104), (for function HFTH); C(201) to C(205), (for subroutine NHBP); C(301) and C(302), (for subroutine EMH); C(401) to C(406), (for subroutine FOFHTH). The above elements are named locally as C101, C102, etc, where the numerical part of the name corresponds to the position in the array C. Elements required by the program, ie, input data, are referenced by local names listed below. (The equivalent array elements are shown in parentheses.)

DRHŌ	(14)	Half width, in km, of twilight transition region.
DRHŌT	(15)	Shift, in km, of transition region, per time step.
EMF	(12)	Ratio of night-to-day electron densities at height of F-layer maximum.
EMŌ	(11)	Ratio of night-to-day electron densities at base of ionospheric model.
ENDL	(6)	Electron density, in electrons/m^3, at top of D-layer model.
ENEL	(8)	Electron density, in electrons/m^3, at maximum of E-layer model.
ENFL	(10)	Electron density, in electrons/m^3, at maximum of F-layer model.
HD	(5)	Height, in km, of top of D-layer model.
HE	(7)	Height, in km, of top (maximum) of E-layer model.
HF	(9)	Height, in km, of maximum of F-layer model.
HŌ	(4)	Base height, in km, of ionospheric model.
RHŌO	(13)	Initial range, in km, to centre of transition region.

```
      SUBROUTINE SETC
C     EVALUATES CONSTANTS FOR THE IONOSPHERIC MODEL FROM THE INPUT DATA
C     VERSION DEF (REVISION 4/11/69 A.R.C. OF VERSION 3A 1/12/67 A.R.C.)
C     ELECTRON DENSITY AS A FUNCTION OF HEIGHT H AND RANGE ANGLE THETA
C     IS GIVEN BY  N(H)*F(H,THETA)
C     N(H) IS A 3-LAYER DAY-TIME IONOSPHERIC MODEL, VIZ. D (QUADRATIC),
C     E (CUBIC) AND F (CUBIC). F(H,THETA) IS A DIMENSIONLESS FACTOR,
C     VALUE UNITY IN DAY REGION, VALUE M(H) (LINEAR) IN NIGHT REGION,
C     AND VARIES AS CUBIC WITH RANGE ANGLE IN TRANSITION REGION. ALL
C     LAYER BOUNDARIES ARE CONCENTRIC WITH EARTH.
C
      COMMON C(100),C101,C102,C103,C104,C105(96),C201,C202,C203,C204,
     1       C205,C206(95),C301,C302,C303(98),C401,C402,C403,C404,C405,
     2       C406
      EQUIVALENCE (C(4),HO),(C(5),HD),(C(6),ENDL),(C(7),HE),(C(8),ENEL),
     1            (C(9),HF),(C(10),ENFL),(C(11),EMO),(C(12),EMF),
     2            (C(13),RHOO),(C(14),DRHO),(C(15),DRHOT)
      DATA RRE(.15698587127159E-03)
C
C     SET MODEL BOUNDARIES IN ASCENDING ORDER IN C(101) ET. SEQ. FOR
C     FUNCTION HFTH
      C101=HO
      C102=HD
      C103=HE
      C104=HF
C     COMPUTE CONSTANTS FOR BASIC PROFILE AND SET IN C(201) ET. SEQ. FOR
C     USE IN SUBROUTINE NHBP
C     D-LAYER. ELECTRON DENSITY VARIES FROM ZERO AT HO TO ENDL AT HD AS
C     N(H) = C(201)*(H-HO)**2
      A1=1./(HD-HO)
      A2=A1*ENDL
      C201=A2*A1
C     E-LAYER. ELECTRON DENSITY VARIES FROM ENDL AT HD TO ENEL AT HE AS
C     N(H) = ENEL-(C(203)+C(202)*(HE-H))*(HE-H)**2
      A1=1./(HE-HD)
      A3=ENEL-ENDL
      A4=A3*A1
      A2=-(A4-A2)*A1*2.
      C202=A2*A1
      C203=A1*A4-A2
C     F-LAYER. ELECTRON DENSITY VARIES FROM ENEL AT HE TO ENFL AT HF AS
C     N(H) = ENFL-(C(205)-C(204)*(HF-H))*(HF-H)**2
      A1=1./(HF-HE)
      A2=ENFL-ENEL
      A3=A2*A1*A1
      C204=A3*A1*2.
      C205=3.*A3
C     COMPUTE CONSTANTS OF M(H), THE RATIO OF NIGHT TO DAY ELECTRON
C     DENSITIES, AND STORE IN C(301) ET. SEQ. FOR USE IN SUBROUTINE FMH
C     M(H) VARIES FROM EMO AT HO TO EMF AT HF AS  M(H) = C(302)+C(301)*H
      A1=1./(HF-HO)
      C302=(EMO*HF-EMF*HO)*A1
      C301=(EMF-EMO)*A1
```

```
C     COMPUTE CONSTANTS OF F(H,THETA) AND STORE IN C(401) ET. SEQ. FOR
C     USE IN SUBROUTINE FOFHTH. F(H,THETA) VARIES FROM UNITY IN DAY
C     REGION TO M(H) IN NIGHT REGION AS  F(H,THETA) = (1+M(H))/2 + OR -
C     (1-M(H))*(C(401)-THETA)*(C(405)-C(406)*(C(401)-THETA)**2)
C     (+ DAY-TO-NIGHT, - NIGHT-TO-DAY) FOR C(402).LT.THETA.LT.C(403)
C     TRANSITION REGION SHIFTS BY C(404) RADIANS PER TIME-STEP
      A1=DRHO*RRE
      C401=RHOO*RRE
      C402=C401-A1
      C403=A1+C401
      C404=DRHOT*RRE
      A1=1./A1
      C405=A1*.75
      C406=.25*A1*A1*A1
      RETURN
      END
```

6.4.6 Subroutine NRX (version NF)

This subroutine calculates the electron density N_e, the refractive index μ, its reciprocal $1/\mu$ and derivatives $\partial\mu/\partial h$, $\partial\mu/\partial\theta$, at the current values of height h and range angle θ.

This version of the subroutine is for use with an ionospheric model where the electron density is given by

$$N_e = N(h).F(h,\theta) \text{ electrons/m}^3,$$

N(h) is a basic (eg, day-time) profile, a function of height only, and is contained in a subroutine NHBP (qv).

$F(h,\theta)$ is a dimensionless function of height and range angle which modifies the basic profile, and is contained in a subroutine FOFHTH (qv).

The refractive index is given by

$$\mu = \sqrt{1 - \frac{0.8061 \times 10^{-10}\, N_e}{f^2}},$$

where f is the signal frequency in MHz and the factor $0.8061 \times 10^{-10}/f^2$ is located in CF(2) (see blank COMMON list).

The derivatives of refractive index are

$$\frac{\partial\mu}{\partial h} = -\frac{0.40305 \times 10^{-10}}{f^2}.\frac{1}{\mu}\left[N(h).\frac{\partial F(h,\theta)}{\partial h} + F(h,\theta).\frac{dN(h)}{dh}\right]$$

and

$$\frac{\partial\mu}{\partial\theta} = -\frac{0.40305 \times 10^{-10}}{f^2}.\frac{1}{\mu}.N(h).\frac{\partial F(h,\theta)}{\partial\theta}.$$

The factor $0.40305 \times 10^{-10}/f^2$ is located in CF(3).

STORAGE LIST

Local Variables

CMU — Value of $-0.40305 \times 10^{-10}/f^2\mu$ where f is the signal frequency in MHz and μ is refractive index.

Blank COMMON

The only items from the list (qv) referred to are elements 2 and 3 of the array CF. These are locally named CF2 and CF3 and contain $0.8061 \times 10^{-10}/f^2$ and $0.40305 \times 10^{-10}/f^2$ respectively.

COMMON/PT/

All items in the list (qv) up to and including DFDTH are referenced with the exception of H and THETA.

```
      SUBROUTINE NRX
C     EVALUATES ELECTRON DENSITY (EN) AND REFRACTIVE INDEX (EMU), ITS
C     RECIPROCAL (RMU) AND DERIVATIVES (DMUDH, DMUDTH) AT CURRENT VALUES
C     HEIGHT (H) AND RANGE ANGLE (THETA)
C     VERSION NF (REVISION 4/11/69 A.R.C. OF SUB RNDX 14/7/67 A.R.C.)
C     ELECTRON DENSITY IS GIVEN BY  N(H)*F(H,THETA)  WHERE N(H) IS A
C     BASIC DAY-TIME IONOSPHERIC PROFILE, MODIFIED IN TRANSITION AND
C     NIGHT-TIME REGIONS BY THE DIMENSIONLESS FACTOR F(H,THETA)
C
      COMMON C(1000),IC(15),CF1,CF2,CF3
      COMMON /PT/ H,THETA,EN,EMU,RMU,DMUDH,DMUDTH,ENH,DNDH,FHTH,DFDH,
     1            DFDTH
C
C     FIND ELECTRON DENSITY (ENH) AND DERIVATIVE (DNDH) IN BASIC PROFILE
      CALL NHBP
C     FIND VALUE (FHTH) AND DERIVATIVES (DFDH, DFDTH) OF MODIFYING TERM
      CALL FOFHTH
C     FORM TOTAL ELECTRON DENSITY (EN)
      EN=ENH*FHTH
C     FORM REFRACTIVE INDEX (EMU) AND RECIPROCAL (RMU)
C     (CF2=0.8061F-10/(F*F))
      EMU=SQRT(1.-EN*CF2)
      RMU=1./EMU
C     FORM DERIVATIVES (DMUDH, DMUDTH) OF REFRACTIVE INDEX
C     (CF3=0.40305E-10/(F*F))
      CMU=-(RMU*CF3)
      DMUDH=(ENH*DFDH+FHTH*DNDH)*CMU
      DMUDTH=ENH*DFDTH*CMU
      RETURN
      END
```

6.4.7 Subroutine NHBP (version DEF/L)

This subroutine calculates the electron density and its derivative at a height h, in a basic ionospheric profile which is a function of height only.

Here the profile consists of D-, E- and F-layer models, the required layer being selected by a control L which takes the values 1(D), 2(E) or 3(F). The layer models are as follows:-

(a) D-layer. A quadratic function of height between heights h_o and h_D such that at height h_o the electron density and its derivative are both zero, and at height h_D the electron density is N_D.

Thus, $$N_e = \frac{N_D}{(h_D - h_o)^2}(h - h_o)^2$$

and $$\frac{dNe}{dh} = 2\frac{N_D}{(h_D - h_o)^2}(h - h_o).$$

The value of h_o is located in C(4) and the factor $N_D/(h_D - h_o)^2$ in C(201).

(b) E-layer. A cubic function of height between heights h_D and h_E such that at height h_D the electron density and its derivative match the D-layer values, and at height h_E the electron density is a maximum of N_E.

Thus, $$N_e = N_E - (h_E - h)^2[a(h_E - h) + b]$$

and $$\frac{dNe}{dh} = (h_E - h)[3a(h_E - h) + 2b],$$

where $$a = \frac{2}{(h_E - h_D)^2}\left(\frac{N_D}{h_D - h_o} - \frac{N_E - N_D}{h_E - h_D}\right)$$

and $$b = \frac{1}{(h_E - h_D)}\left[\frac{3(N_E - N_D)}{h_E - h_D} - \frac{2N_D}{h_D - h_o}\right].$$

The value of h_E is located in C(7), that of N_E in C(8) and the constants a and b in C(202) and C(203) respectively.

(c) F-layer. A cubic function of height between heights h_E and h_F such that at height h_E the electron density and its derivative (zero) match the E-layer values, and at height h_F the electron density is a maximum of N_F.

Thus, $$N_e = N_F - (h_F - h)^2[b - a(h_F - h)]$$

and $$\frac{dN_e}{dh} = (h_F - h)[2b - 3a(h_F - h)],$$

where $$a = \frac{2(N_F - N_E)}{(h_F - h_E)^3}$$

and $$b = \frac{3(N_F - N_E)}{(h_F - h_E)^2}.$$

The value of h_F is located in C(9), that of N_F in C(10) and the constants a and b in C(204) and C(205) respectively.

STORAGE LIST

Local Variables

A)
) Intermediate variables of various definitions.
B)

HM Difference between the input height and the reference height of the particular ionospheric layer.

Blank COMMON

C Here the array is segmented so that relevant elements may be referenced without subscripting. The required elements are as follows (the numerical part of a name indicates the position in the original array, ie, C201 ≡ C(201)):-

C4	Base height, in km, of the ionospheric model.
C7	Height, in km, of the maximum of the E-layer model.
C8	Maximum electron density, electrons/m^3, in the E-layer model.
C9	Height, in km, of the maximum of the F-layer model.
C10	Maximum electron density, electrons/m^3, in the F-layer model.
C201	A constant of the D-layer model.
C202, C203	Constants of the E-layer model.
C204, C205	Constants of the F-layer model.

IC Only element 15 of the array is used and is referenced by the equivalent name:-

L Control indicating the required ionospheric layer model, ie, 1 for D, 2 for E or 3 for F.

No other items in the list are required.

COMMON/PT/

The only items in the list (qv) which are referenced here are H, ENH and DNDH.

```
      SUBROUTINE NHBP
C     CALCULATES ELECTRON DENSITY (ENH) AND DERIVATIVE (DNDH) AT HEIGHT
C     H IN BASIC IONOSPHERIC PROFILE N(H)
C     VERSION DEF/L (REVISION 4/11/69 A.R.C. OF SUB NHP 1/12/67 A.R.C.)
C     3-LAYER PROFILE - D (QUADRATIC), E (CUBIC) AND F (CUBIC)
C     LAYER SELECTION BY NUMBER (L)
C
      COMMON C(3),C4,C5,C6,C7,C8,C9,C10,C11(190),C201,C202,C203,C204,
     1       C205,C206(795),IC(15)
      COMMON /PT/ H,THETA,EN,EMU,RMU,DMUDH,DMUDTH,ENH,DNDH
      EQUIVALENCE (IC(15),L)
C
C     SELECT REQUIRED LAYER
      GO TO (1,2,3),L
C     D-LAYER. ELECTRON DENSITY VARIES FROM ZERO AT HEIGHT C(4) TO C(6)
C     AT HEIGHT C(5) AS  N(H)=C(201)*(H-C(4))**2
    1 HM=H-C4
      A=HM*C201
      ENH=A*HM
      DNDH=A+A
      RETURN
C     E-LAYER. ELECTRON DENSITY VARIES FROM C(6) AT HEIGHT C(5) TO C(8)
C     AT HEIGHT C(5) AS  N(H)=C(8)-(C(203)+C(202)*(C(7)-H))*(C(7)-H)**2
    2 HM=C7-H
      A=HM*C202
      B=(A+C203)*HM
      ENH=C8-B*HM
      DNDH=A*HM+B+B
      RETURN
C     F-LAYER. ELECTRON DENSITY VARIES FROM C(8) AT HEIGHT C(7) TO C(10)
C     AT HEIGHT C(9) AS  N(H)=C(10)-(C(205)-C(204)*(C(9)-H))*(C(9)-H)**2
    3 HM=C9-H
      A=HM*C204
      B=(A-C205)*HM
      ENH=B*HM+C10
      DNDH=-(A*HM+B+B)
      RETURN
      END
```

6.4.8 Subroutine SETC (version DEF/ES)

This subroutine calculates various constants from the input ionospheric model data and stores them in the blank COMMON array C for later use.

This version is similar to subroutine SETC (version DEF), the difference being the addition of a sporadic E-layer to the ionospheric model. The sporadic E-layer function is defined by

$$N_{ES}(h) = \hat{N}_{ES} e^{\left[-2\left(\frac{h-h_{ES}}{W_{ES}}\right)^2\right]},$$

where $\hat{N}_{ES}$ is the maximum electron density in the layer, input in C(17), h_{ES} is the height of the peak, input in C(16), and W_{ES} is the layer half-width, input in C(18) (ie, half the height difference between points where the function is down to e^{-2} of peak value).

The function, which is added to the ionospheric model, is obviously non-zero for all heights, but for the purposes of defining a region where a change in integration step-length is required, ie, where the electron density is changing significantly faster than that in the remainder of the model at similar heights, the boundaries are set at levels where the value of the function is approximately 1% of the peak value, ie, at $h_{ES} \pm 1.5W_{ES}$. (The limits within which the function is assumed to contribute to the overall electron density are set wider than the above, see subroutine NHES.)

The ordering of the various boundary heights for function HFTH (qv) is less simple than in version DEF, as it is assumed that the sporadic E model may be arbitrarily placed on the basic (D - E - F) profile, with the restriction that the sporadic E model is sufficiently narrow for its boundaries (1%) to lie in the same or adjacent layers of the basic profile.

The constants required by subroutine NHES are:-

$\sqrt{2}/W_{ES}$ set in C(501)

and $2\sqrt{2}/W_{ES}$ set in C(502).

The remainder of the program, concerning the basic profile, night-time and transition regions, is identical to version DEF.

The form of the sporadic E-layer model is shown in figure 9, where the data of example 2, section 7, are used.

STORAGE LIST

Local Variables

A1, A2, A3, A4 — Intermediate variables of various definitions.

DHES — Difference between height of the peak of the function defining the sporadic E electron density and a height where the value of the function is approximately 1% of the peak, ie, 1.5 times the function half width, the latter being defined at the e^{-2} level.

HESL — Lower height, in km, where the sporadic E function is approximately 1% of peak value.

HESU — Upper height, in km, where the sporadic E function is approximately 1% of peak value.

RŌŌT2 — Square root of 2, set in DATA statement.

RRE — Reciprocal of earth radius (6370 km), set in DATA statement.

Blank CŌMMŌN Variables

C — Here the array is segmented so that relevant elements may be referenced without subscripting. The elements defined in this program are: C(101) to C(106), (for function HFTH); C(201) to C(205), (for subroutine NHBP); C(301) and C(302), (for subroutine EMH); C(401) to C(406), (for subroutine FŌFHTH); C(501) and C(502), (for subroutine NHES). The above elements are named locally as C101, C102, etc, where the numerical part of the name corresponds to the position in the array C. Elements required by the program, ie, input data, are referenced by local names listed below (the equivalent array elements are shown in parentheses).

DRHŌ	(14)	Half width, in km, of twilight transition region.
DRHŌT	(15)	Shift, in km, of transition region per time-step.
EMF	(12)	Ratio of night-to-day electron densities at height of F-layer maximum.
EMŌ	(11)	Ratio of night-to-day electron densities at base of ionospheric model.
ENDL	(6)	Electron density, in electrons/m^3, at top of D-layer model.
ENEL	(8)	Electron density, in electrons/m^3, at maximum of E-layer model.

ENESL	(17)	Electron density, in electrons/m^3, at peak of sporadic E-layer model.
ENFL	(10)	Electron density, in electrons/m^3, at maximum of F-layer model.
HD	(5)	Height, in km, of top of D-layer model.
HE	(7)	Height, in km, of top (maximum) of E-layer model.
HES	(16)	Height, in km, of peak of sporadic E-layer model.
HF	(9)	Height, in km, of maximum of F-layer model.
$H\bar{O}$	(4)	Base height, in km, of ionospheric model.
$RH\bar{O}O$	(13)	Initial range, in km, to centre of transition region.
WES	(18)	Half width, in km, of sporadic E-layer model (defined at e^{-2} level).

```
      SUBROUTINE SETC
C     EVALUATES CONSTANTS FOR THE IONOSPHERIC MODEL FROM THE INPUT DATA
C     VERSION DEF/ES (ORIG. R.M.J. REVISED 20/10/69 A.R.C.)
C     ELECTRON DENSITY AS A FUNCTION OF HEIGHT H AND RANGE ANGLE THETA
C     IS GIVEN BY  N(H)*F(H,THETA) + NES(H)
C     N(H) IS A 3-LAYER DAY-TIME IONOSPHERIC MODEL, VIZ. D (QUADRATIC),
C     E (CUBIC) AND F (CUBIC). F(H,THETA) IS A DIMENSIONLESS FACTOR,
C     VALUE UNITY IN DAY REGION, VALUE M(H) (LINEAR) IN NIGHT REGION,
C     AND VARIES AS CUBIC WITH RANGE ANGLE IN TRANSITION REGION. NES(H)
C     IS A SPORADIC-E MODEL (GAUSSIAN). ALL LAYER BOUNDARIES ARE
C     CONCENTRIC WITH EARTH.
C
      COMMON C(100),C101,C102,C103,C104,C105,C106,C107(94),C201,C202,
     1       C203,C204,C205,C206(95),C301,C302,C303(98),C401,C402,C403,
     2       C404,C405,C406,C407(94),C501,C502
      EQUIVALENCE (C(4),HO),(C(5),HD),(C(6),ENDL),(C(7),HE),(C(8),ENFL),
     1            (C(9),HF),(C(10),ENFL),(C(11),FMO),(C(12),FMF),
     2            (C(13),RHOO),(C(14),DRHO),(C(15),DRHOT),(C(16),HES),
     3            (C(17),ENESL),(C(18),WES)
      DATA RRE(.15698587127159E-3),ROOT2(1.4142135623731)
C
C     THE (PSEUDO)BOUNDARIES OF THE SPORADIC-E LAYER ARE SET AT HEIGHTS
C     WHERE ELECTRON DENSITY IS APPROX .01 OF PEAK VALUE (ENESL)
C     I.E. AT HES + OR - 1.5*WES
      DHES=1.5*WES
      HESU=DHES+HES
      HESL=HES-DHES
C     SET MODEL BOUNDARIES IN ASCENDING ORDER IN C(101) ET. SEQ. FOR
C     FUNCTION HETH. (IT IS ASSUMED THAT THE SPORADIC-E BOUNDARIES LIE
C     IN THE SAME OR ADJACENT LAYERS OF THE BASIC PROFILE)
      C101=HO
      IF(HD-HESL)4,1,1
```

```
      1 C102=HESL
        IF(HD-HESU)3,2,2
      2 C103=HESU
        C104=HD
        GO TO 7
      3 C103=HD
        GO TO 6
      4 C102=HD
        IF(HE-HESL)9,5,5
      5 C103=HESL
        IF(HE-HESU)8,6,6
      6 C104=HESU
      7 C105=HE
        GO TO 11
      8 C104=HE
        GO TO 10
      9 C103=HE
        C104=HESL
     10 C105=HESU
     11 C106=HF
C       COMPUTE CONSTANTS FOR BASIC PROFILE AND SET IN C(201) ET. SEQ. FOR
C       USE IN SUBROUTINE NHEP
C       D-LAYER. ELECTRON DENSITY VARIES FROM ZERO AT HO TO ENDL AT HD AS
C       N(H) = C(201)*(H-HO)**2
        A1=1./(HD-HO)
        A2=A1*ENDL
        C201=A2*A1
C       E-LAYER. ELECTRON DENSITY VARIES FROM ENDL AT HD TO ENEL AT HE AS
C       N(H) = ENEL-(C(203)+C(202)*(HE-H))*(HE-H)**2
        A1=1./(HE-HD)
        A3=ENEL-ENDL
        A4=A3*A1
        A2=-(A4-A2)*A1*2.
        C202=A2*A1
        C203=A1*A4-A2
C       F-LAYER. ELECTRON DENSITY VARIES FROM ENEL AT HE TO ENFL AT HF AS
C       N(H) = ENFL-(C(205)-C(204)*(HF-H))*(HF-H)**2
        A1=1./(HF-HE)
        A2=ENFL-ENEL
        A3=A2*A1*A1
        C204=A3*A1*2.
        C205=3.*A3
C       COMPUTE CONSTANTS OF M(H), THE RATIO OF NIGHT TO DAY ELECTRON
C       DENSITIES, AND STORE IN C(301) ET. SEQ. FOR USE IN SUBROUTINE EMH
C       M(H) VARIES FROM EMO AT HO TO EMF AT HF AS M(H) = C(302)+C(301)*H
        A1=1./(HF-HO)
        C302=(EMO*HF-EMF*HO)*A1
        C301=(EMF-EMO)*A1
C       COMPUTE CONSTANTS OF F(H,THETA) AND STORE IN C(401) ET. SEQ. FOR
C       USE IN SUBROUTINE FOFHTH. F(H,THETA) VARIES FROM UNITY IN DAY
C       REGION TO M(H) IN NIGHT REGION AS F(H,THETA) = (1+M(H))/2 + OR -
C       (1-M(H))*(C(401)-THETA)*(C(405)-C(406)*(C(401)-THETA)**2)
C       (+ DAY-TO-NIGHT, - NIGHT-TO-DAY) FOR C(402).LT.THETA.LT.C(403)
C       TRANSITION REGION SHIFTS BY C(404) RADIANS PER TIME STEP
```

```
      A1=DRHO*RRE
      C401=RHOC*RRE
      C402=C401-A1
      C403=A1+C401
      C404=DRHOT*RRE
      A1=1./A1
      C405=A1*.75
      C406=.25*A1*A1*A1
C     COMPUTE CONSTANTS FOR SPORADIC-E LAYER NES(H) AND STORE IN C(501)
C     ET. SEQ. FOR USE IN SUBROUTINE NHES. THE MODEL IS OF THE FORM
C     NES(H) = ENESL*EXP(-2*((H-HES)/WES)**2)
      C501=ROOT2/WES
      C502=C501+C501
      RETURN
      END
```

6.4.9 Subroutine NRX (version NF + ES)

This subroutine calculates the electron density N_e, the refractive index μ, its reciprocal $1/\mu$ and derivatives $\partial\mu/\partial h$, $\partial\mu/\partial\theta$, at the current values of height h and range angle θ.

This version of the subroutine is for use with an ionospheric model where the electron density is given by

$$N_e = N(h).F(h,\theta) + N_{ES}(h) \text{ electrons/m}^3,$$

N(h) is a basic profile (eg, day-time), a function of height only, and is contained in a subroutine NHBP (qv).

$F(h,\theta)$ is a dimensionless function of height and range angle which modifies the basic profile and is contained in a subroutine FOFHTH (qv).

$N_{ES}(h)$ is a model of a sporadic E-layer, a function of height only, and is contained in a subroutine NHES (qv).

The refractive index is given by

$$\mu = \sqrt{1 - \frac{0.8061 \times 10^{-10} N_e}{f^2}},$$

where f is the signal frequency in MHz and the factor $0.8061 \times 10^{-10}/f^2$ is located in CF(2) (see blank COMMON list).

The derivatives of refractive index are

$$\frac{\partial\mu}{\partial h} = -\frac{0.40305 \times 10^{-10}}{f^2}.\frac{1}{\mu}.\left[N(h).\frac{\partial F(h,\theta)}{\partial h} + F(h,\theta).\frac{dN(h)}{dh} + \frac{dN_{ES}(h)}{dh}\right]$$

and

$$\frac{\partial\mu}{\partial\theta} = -\frac{0.40305 \times 10^{-10}}{f^2}.\frac{1}{\mu}.N(h).\frac{\partial F(h,\theta)}{\partial\theta}.$$

The factor $0.40305 \times 10^{-10}/f^2$ is located in CF(3) (see blank COMMON list).

STORAGE LIST

Local Variables

CMU — Value of $-0.40305 \times 10^{-10}/f^2\mu$, where f is the signal frequency in MHz and μ is refractive index.

Blank COMMON

The only items from the list (qv) referred to here are elements 2 and 3 of the array CF. These are locally named CF2 and CF3 and contain $0.8061 \times 10^{-10}/f^2$ and $0.40305 \times 10^{-10}/f^2$ respectively.

COMMON/PT/

All items in the list (qv) are used with the exception of DMDH, EM, H and THETA.

```
      SUBROUTINE NRX
C     EVALUATES ELECTRON DENSITY (EN) AND REFRACTIVE INDEX (EMU), ITS
C     RECIPROCAL (RMU) AND DERIVATIVES (DMUDH, DMUDTH) AT CURRENT VALUES
C     OF HEIGHT (H) AND RANGE ANGLE (THETA)
C
C     VERSION NE+ES (REVISED 16/10/69 A.R.C.)
C     ELECTRON DENSITY IS GIVEN BY  N(H)*F(H,THETA) + NES(H)
C     WHERE N(H) IS A BASIC DAY-TIME IONOSPHERIC PROFILE, MODIFIED IN
C     TRANSITION AND NIGHT-TIME REGIONS BY THE DIMENSIONLESS FUNCTION
C     F(H,THETA), AND NES(H) IS A SPORADIC-E ENHANCEMENT
C
      COMMON C(1000),IC(15),CF1,CF2,CF3
      COMMON /PT/ H,THETA,EN,EMU,RMU,DMUDH,DMUDTH,ENH,DNDH,FHTH,DFDH,
     1            DFDTH,EM,DMDH,ENS,DNSDH
C     FIND ELECTRON DENSITY (ENH) AND DERIVATIVE (DNDH) IN BASIC PROFILE
      CALL NHBP
C     FIND VALUE (FHTH) AND DERIVATIVES (DFDH,DFDTH) OF MODIFYING TERM
      CALL FOFHTH
C     FIND ELECTRON DENSITY (ENS) AND DERIVATIVE (DNSDH) IN SPORADIC-E
      CALL NHES
C     FORM TOTAL ELECTRON DENSITY (EN)
      EN=ENH*FHTH+ENS
C     FORM REFRACTIVE INDEX (EMU) AND RECIPROCAL (RMU)
C     (CF2=0.8061E-10/F**2)
      EMU=SQRT(1.-EN*CF2)
      RMU=1./EMU
C     FORM DERIVATIVES (DMUDH, DMUDTH) OF REFRACTIVE INDEX
C     (CF3=0.40305E-10/F**2)
      CMU=-(RMU*CF3)
      DMUDH=(ENH*DFDH+FHTH*DNDH+DNSDH)*CMU
      DMUDTH=ENH*DFDTH*CMU
      RETURN
      END
```

6.4.10 Subroutine NHBP (version DEF/H)

This subroutine calculates the electron density and its derivative at a height h in a basic ionospheric profile which is a function of height only.

This version is for the same profile as that described in version DEF/L (qv), but is intended for use with a model containing a sporadic E-layer, and as the latter may be arbitrarily positioned, the selection of the appropriate layer function in the basic profile is done by height comparison instead of direct selection by layer number.

STORAGE LIST

Local Variables

A	) Intermediate variables of various definitions.
B	)
HM	Difference between the input height and the reference height of a particular ionospheric layer.

Blank COMMON

C — This array is segmented here so that relevant elements may be referenced without subscripting. The elements used are as follows, the numerical part of the local name indicating the position in the original array, ie, C201 ≡ C(201):-

C4	Base height, in km, of the ionospheric model.
C5	Height, in km, of the top of the D-layer model.
C7	Height, in km, of the top (maximum) of the E-layer model.
C8	Maximum electron density, electrons/m^3, in the E-layer model.
C9	Height, in km, of the maximum of the F-layer model.
C10	Maximum electron density, electrons/m^3, in the F-layer model.
C201	A constant of the D-layer model.
C202, C203	Constants of the E-layer model.
C204, C205	Constants of the F-layer model.

No other items in the list are used.

COMMON/PT/

The only items in the list (qv), which are referenced here are DNDH, ENH and H.

```
      SUBROUTINE NHBP
C     BASIC IONOSPHERIC PROFILE N(H)
C     VERSION DEF/H (ORIG. R.M.J. REVISED 17/10/69 A.P.C.)
C     3-LAYER PROFILE - D (QUADRATIC), E (CUBIC) AND F (CUBIC)
C     LAYER SELECTION BY HEIGHT (H)
C
      COMMON C(3),C4,C5,C6,C7,C8,C9,C10,C11(190),C201,C202,C203,C204,
     1       C205
      COMMON /PT/ H,THETA,EN,EMU,RMU,DMUDH,DMUDTH,ENH,DNDH
C     LOCATE RELEVANT LAYER (C5 IS D-E BOUNDARY, C7 IS E-F BOUNDARY)
      IF(H-C5)2,2,1
    1 IF(H-C7)3,4,4
C     D-LAYER. ELECTRON DENSITY VARIES FROM ZERO AT HEIGHT C4 TO C6 AT
C     HEIGHT C5 AS N(H) = C201*(H-C4)**2
    2 HM=H-C4
      A=HM*C201
      ENH=A*HM
      DNDH=A+A
      RETURN
C     E-LAYER. ELECTRON DENSITY VARIES FROM C6 AT HEIGHT C5 TO C8 AT
C     HEIGHT C7 AS N(H) = C8-(C203+C202*(C7-H))*(C7-H)**2
    3 HM=C7-H
      A=HM*C202
      B=(A+C203)*HM
      ENH=C8-B*HM
      DNDH=A*HM+B+B
      RETURN
C     F-LAYER. ELECTRON DENSITY VARIES FROM C8 AT HEIGHT C7 TO C10 AT
C     HEIGHT C9 AS N(H) = C10-(C205-C204*(C9-H))*(C9-H)**2
    4 HM=C9-H
      A=HM*C204
      B=(A-C205)*HM
      ENH=B*HM+C10
      DNDH=-(A*HM+B+B)
      RETURN
      END
```

6.4.11 Subroutine NHES

This subroutine calculates the electron density and its derivative at height h in a sporadic E-layer model which is a function of height only.

In this version of the subroutine, the sporadic E-layer is represented by

$$N_{ES}(h) = \hat{N}_{ES} \exp\left[- 2\left(\frac{h - h_{ES}}{W_{ES}}\right)^2 \right]$$

so that $$\frac{dN_{ES}(h)}{dh} = - \frac{4(h - h_{ES})}{W_{ES}^2} . N_{ES}(h),$$

$\hat{N}_{ES}$ is the peak electron density in the layer, input in C(17), at height h_{ES}, input in C(16), and W_{ES} is the layer "half-width", input in C(18), ie, the difference between the peak height and a height where the electron density has fallen to e^{-2} of the peak value.

The layer is assumed to make negligible contribution to the overall ionospheric model when the magnitude of $\sqrt{2}(h - h_{ES})/W_{ES}$ exceeds 6.

The constant $\sqrt{2}/W_{ES}$ is located in C(501) and $2\sqrt{2}/W_{ES}$ is in C(502). (See notes on subroutine SETC, version DEF/ES.)

7. SUMMARY OF INPUT DATA REQUIRED

A list of the data to be input, in the required order, is given below. Apart from a brief statement for each set of data, descriptions of the various items are not provided, but the relevant COMMON list is referenced, eg, /PLOT/ for COMMON/PLOT/ list, // for blank COMMON lists. Data not so referenced are local to subroutine INDATA and their descriptions are to be found in the storage list for the subroutine. In many cases, possible values, including maxima and/or minima where appropriate, are given, but the absence of such information does not imply any lack of restriction in value.

Each set of data corresponds to one READ statement, and the items in each set are in consecutive fields according to the FORMAT indicated.

Note: As the program uses the AWRE SC4060 graph plotter, a data card to be read by the library subroutine SCLIBR must precede the cards containing the data listed below (see relevant CAN notes).

(1) FORMAT 12I5 — Run controls.

JOBNO	Max. of 4 digits	/IMO/
NTF	Min. 1	/IMO/
NTL	≥ NTF	/IMO/
NDT	Min. 1	/IMO/
NF	Min. 1, max. 24	/IMO/
NH	Min. 1, max. 5	/IMO/
KA	1, 2 or 3 (see data set 11)	/IMO/
KP	1, 2 or 3	/IMO/
KG	1 or 2	/IMO/
KX	1, 2, 3, 4, 5 or 6	/IMO/
JP	1 or 2	/IMO/
JX	1 or 2	/IMO/

(2) FORMAT 7I5 — Output options

MPT	1 or 2	/IMO/
MPLT	1 or 2	/IMO/
MPRNT	1 or 2	/IMO/
MPUN	1 or 2	/IMO/
MXPG	1 or 2	/IMO/
MSIG	1 or 2	/IMO/
MXDAT	1 (<u>if</u> MXPG = 1 <u>and</u> MSIG = 1) or 2	/IMO/

(3) FORMAT 16I5 — Ionosphere model controls

NC	Min. 3	
NFMI	Max. 100	
NIC	Min. 2, max. 14	
IC(1)	Max. of 3 digits	//
IC(2)	Min. 2, max. 17	//
IC(3))		
.....)	As required	//
IC(NIC))		

STORAGE LIST

Local Variables

A — Value of $(h - h_{ES})\sqrt{2}/W_{ES}$, where h is current height, h_{ES} is height of peak of sporadic E-layer model and W_{ES} is "half-width" of sporadic E-layer.

Blank COMMON

C — Elements 16, 17, 501 and 502 of the array are used here and are referenced by the following local names:-

C16 — Height, in km, of the peak of the sporadic E-layer model.

C17 — Maximum electron density, electrons/m^3, of the sporadic E-layer model.

C501 — Value of $\sqrt{2}/W_{ES}$, where W_{ES} is the "half-width" of the sporadic E-layer model.

C502 — Value of $2\sqrt{2}/W_{ES}$.

No other items in the list are used.

COMMON/PT/

The only items in the list (qv), which are referenced here are DNSDH, ENS and H.

```
      SUBROUTINE NHES
C     MODEL OF A SPORADIC-E LAYER NES(H)
C     VERSION A (7/11/68 R.M.J. REVISED 17/10/69 A.R.C.)
C     MODEL IS OF THE FORM NES(H) = NESMAX*EXP(-2.*((H-HES)/WES)**2)
C
      COMMON C(15),C16,C17,C18(483),C501,C502
      COMMON /PT/ H,THETA,EN,FMU,RMU,DMUDH,DMUDTH,ENH,DNDH,FHTH,DFDH,
     1            DFDTH,FM,DMDH,ENS,DNSDH
C     CHECK IF SPORADIC-E IS SIGNIFICANT AT HEIGHT H, I.E. IF ABS(H-HES)
C     LESS THAN APPROX 4.*WES. (HES=C17, SQRT(2.)/WES=C501)
      A=(H-C16)*C501
      IF(ABS(A)-6.)1,2,2
C     FORM ELECTRON DENSITY (ENS) AND ITS DERIVATIVE (DNSDH) IN
C     SPORADIC-E LAYER. (NESMAX=C17, 2.*SQRT(2.)/WES=C502)
    1 ENS=EXP(-A*A)*C17
      DNSDH=-(ENS*A*C502)
      RETURN
C     SPORADIC-E LAYER HAS NEGLIGIBLE EFFECT AT THIS HEIGHT
    2 ENS=0.
      DNSDH=0.
      RETURN
      END
```

Note: If KA = 1, only one set of data type 11 is required.
If KA = 2, 1 + (NTL - NTF)/NDT sets of data type 11 are required (on separate cards), ordered:-

1st time-step,
2nd time-step,
etc.

If KA = 3, (1 + (NTL - NTF)/NDT)*NF sets of data type 11 are required (on separate cards), ordered:-

1st time-step, 1st frequency,
2nd frequency,
etc,

2nd time-step, 1st frequency,
2nd frequency,
etc,

etc.

Examples of data decks for two typical types of run, using the ionospheric models included with the program, are described below.

(4)	FŌRMAT 8E10.5		Ionosphere model data	
	C(1)	Usually zero		//
	C(2)	Usually zero		//
	C(3)			//
	C(4)	)		//
	.....	) As required		
	C(NC)	)		
(5)	FŌRMAT 10A8		Ionosphere data printout FŌRMAT	
	FMI(1)			
	.....			
	FMI(NFMI)			
(6)	FŌRMAT 8F10.6		Step lengths	
	ST(1)			//
	ST(2)	)		
	.....	) As required. NS = IC(2)-1		//
	ST(NS)	)		
(7)	FŌRMAT 3E10.5		Ray limits and mode split	
	HM			//
	RHŌM			
	HEF			/IMŌ/
(8)	FŌRMAT 8F10.6		Signal frequencies	
	FREQ(1)			/IMŌ/
	FREQ(2)	)		
	.....	) As required		/IMŌ/
	FREQ(NF)	)		
(9)*	FŌRMAT 4I5		Plotting controls	
	NTYP	1, 2 or 3		/PLŌT/
	NTITLE	Max. 400		/PLŌT/
	MAXH	Multiple (≯10) of INCDH		/PLŌT/
	INCDH	≤ MAXH/2		/PLŌT/
(10)*	FŌRMAT 10A8		Title for plotted output	
	TITLE(1)			/PLŌT/
	.....			
	TITLE(NTITLE)			

*9 and 10 are present <u>only if</u> MPLT = 1

(11)	FŌRMAT I8,2F8.3		Takeoff angle data	
	NA	Min. 1, max. 100		/IMŌ/
	ALPH1	Min. zero		/IMŌ/
	DA	Positive, ALPH1 + DA(NA - 1) < 90°		/IMŌ/

IONOSPHERE MODEL USED IN EXAMPLE 1.
****

IONOSPHERE DATA

MODEL 301 4 BOUNDARIES

TRANSITION TYPE 2 (1 DAY-TO-NIGHT, 2 NIGHT-TO-DAY)

IONO C OF C (.0, .0) BASE RADIUS 6430.0KM

DAY PROFILE	HEIGHT	DENSITY
	60.0	.0000E+00
D-LAYER		
	85.0	.2500E+10
E-LAYER		
	110.0	.1000E+12
F-LAYER		
	300.0	.1000E+13

NIGHT/DAY DENSITY RATIO - AT BASE .0, AT F-MAX .3

TRANSITION REGION - RANGE TO CENTRE (LT=1) -1000.0KM
HALF WIDTH 1000.0KM
SHIFT/TIME-STEP 500.0KM

DATA DECK FOR EXAMPLE 1.

LINES MARKED WITH * FORM THE ACTUAL DATA DECK. ALL OTHERS ARE COMMENTS ONLY.
NUMBERS ON LEFT CORRESPOND WITH SECTION NUMBERS IN SUMMARY.

```
COL          1         2         3         4         5         6         7         8
    12345678901234567890123456789012345678901234567890123456789012345678901234567890

  * 'SC4060 DATA CARD ACCORDING TO LOCAL INSTRUCTIONS'

 1. JOBNO  NTF  NTL  NDT   NF   NH   KA   KP   KG   KX   JP   JX
  *   307    1    8    7    1    5    2    1    1    1    1    1

 2.   MPT     MPRNT      MXPG     MXDAT
         MPLT      MPUN      MSIG
  *     2    1    1    2    2    2    2

 3.    NC       NIC     IC(2)
         NFMI     IC(1)     IC(3)
  *    15   70    3  301    4    2
```

7.1 Example 1 Run number 307

(a) Ionosphere model

Three-layer model with movable twilight region, contained in subprograms SETC (version DEF), TVP, HFTH, NRX (version NF), NHBP (version DEF/L), FŌFHTH and EMH. The data used (IC(1) to IC(NIC), NIC = 3, and C(1) to C(NC), NC = 15) are given below, printed out according to the format entered in array FMI (contained on 7 cards, NFMI = 70).

(b) Model states (time-steps)

(i) Day-time, obtained with the transition (night-to-day) in its initial position behind the transmitter, ie, NTF = 1.

(ii) Night-to-day twilight region centred 2500 km from the transmitter, a shift of 3500 km from the initial position (-1000 km), obtained by an increment in time-step number NDT = 7 (ie, 3500/500) to NTL = 8.

(c) Signal frequencies (FREQ)

13 MHz (NF = 1).

(d) Ray takeoff angles

All angles from 0° (ALPH1) to escape, in increments of 1° (DA), for both time-steps. From previous tests, first escape angle in the day-time state is about 41° (NA = 42), and in the twilight (2500 km) state, about 15° (NA = 16). Since the angle data are different for the two time-steps, KA = 2 (or 3 as only one frequency).

(e) Ray limits

Up to 5 hops (NH) or 15000 km maximum range (RHOM). Maximum height (HM) less than 300 km (eg, 299 km. If set to 300 km, ie, top boundary of model, there may be an unnecessary call to subroutine LINE2B).

(f) Step-lengths (ST)

10 km in all three layers.

(g) Outputs

(i) Ray-path plots (MPLT = 1), on both hard copy and microfilm (NTYP = 3), with height scale to 300 km (MAXH) marked at 50 km intervals (INCDH). Description (TITLE) contained on 3 cards (NTITLE = 30).

(ii) Summary printout of ray-hop data (MPRNT = 1) with phase and group paths in km (KP = KG = 1).

No other outputs (MPT, MPUN, MXPG, MSIG and MXDAT all 2) so KX, JP, JX and HEF are not relevant, but are given reasonable values for completeness.

7.2 Example 2 Run number 297

(a) Ionosphere model

As in example 1, but including the sporadic E-layer model. Subroutines SETC (version DEF/ES), NRX (version NF + ES) and NHBP (version DEF/H) replace the previous versions used, and subroutine NHES is required in addition. The data used (IC(1) to IC(NIC), NIC = 3, and C(1) to C(NC), NC = 18) are given below, printed out according to the format entered in array FMI (contained on 8 cards, NFMI = 80).

(b) Model states (time-steps)

Day-time only, obtained with the transition (night-to-day) in its initial position behind the transmitter, ie, NTF = 1, NDT = 1, NTL = 1.

(c) Signal frequencies (FREQ)

16, 17 and 18 MHz (NF = 3).

(d) Ray takeoff angles

All angles from 0° (ALPH1) to escape, in increments of 0.5° (DA), for all frequencies. From previous tests, first escape angle in day-time state at 16 MHz is about 30° (NA = 61), at 17 MHz, about 27.5° (NA = 56) and at 18 MHz, about 25° (NA = 51). Since the angle data are different for each frequency, KA = 3.

(e) Ray limits

Single hop only (NH = 1) or 6000 km maximum range (RHŌM). Maximum height (HM) less than 300 km (eg, 299 km, see note for example 1).

(f) Step-lengths (ST)

10 km in D and F regions, 5 km in E region and 0.1 km in sporadic E region in height bracket $h_{ES} \pm 1.5W_{ES}$, ie, 98.5 to 101.5 km. Note: As this bracket lies totally within the E region, successive "layers" are D, E, E_S, E and F and corresponding step-lengths are 10, 5, 0.1, 5 and 10 km.

(g) Outputs

(i) Summary printout of ray-hop data (MPRNT = 1), with phase in cycles (KP = 3) and group time in ms (KG = 2).

(ii) "RAYSET" cards (MPUN = 1). Since the rays are single hop, the value of JP (1 or 2) does not affect the order of the cards.

(iii) Calculations of excess phase in cycles and excess group time in μs (KX = 6).

```
4. ARRAY C, ELEMENTS 1 TO NC
    IONO-CENTRE-COORDS  BASE-RAD  H-OF-BASE  H-D-TOP  NE-D-TOP   H-E-MAX  NE-E-MAX
 *  .0000E+00 .0000E+00 6430.E+00 60.00E+00 85.00E+00 2.500E+09 110.0E+00 1.000E+11
     H-F-MAX  NE-F-MAX     -NIGHT/DAY-NE-      --NIGHT-TO-DAY-TRANSITION--
                            BASE      F-MAX     CENTRE  HALF-WIDTH SHIFT/TS
 *  300.0E+00 1.000E+12 .0000E+00 .3000E+00-1000.E+00 1000.E+00 500.0E+00

5. ARRAY FMI - FORMAT TO PRINT IC AND C
 * (20H1    IONOSPHERE DATA/5X15H---------------//5X5HMODEL,I4,I8,11H BOUNDARIES//5
 * X15HTRANSITION TYPE,I2,33H (1 DAY-TO-NIGHT, 2 NIGHT-TO-DAY)///5X13HIONO C OF C (
 * ,F3.1,1H,,F3.1,16H)     BASE RADIUS,F7.1,2HKM///5X11HDAY PROFILE,7X6HHEIGHT,5X7HD
 * ENSITY//F29.1,4X9H.0000E+00/9X7HD-LAYER/F29.1,F13.4/9X7HE-LAYER/F29.1,F13.4/9X7H
 * F-LAYER/F29.1,F13.4///5X33HNIGHT/DAY DENSITY RATIO - AT BASE,F3.1,10H, AT F-MAX,
 * F3.1///5X42HTRANSITION REGION - RANGE TO CENTRE (LT=1),F8.1,2HKM/37X10HHALF WIDT
 * H,F8.1,2HKM/32X15HSHIFT/TIME-STEP,F8.1,2HKM////)

6. ARRAY ST - STEP LENGTHS IN EACH LAYER
      IN-D       IN-E       IN-F
 *  10.000000 10.000000 10.000000

7.     HM        RHOM        HEF
 *  299.0E+00 1.500E+04 110.0E+00

8. ARRAY FREQ - SIGNAL FREQUENCIES
 *  13.000000

9.     NTITLE     INCDH
    NTYP      MAXH
 *     3   30  300   50

10. ARRAY TITLE - EXAMPLE OF TITLE FOR PLOTTED OUTPUT
 * ILLUMINATION CHECK RUN  12/1/70
 *                                  LT=1 DAY-TIME
 *                                  LT=8 NIGHT-TO-DAY CENTRED AT 2500KM

11.        NA   ALPH1      DA    ANGLE DATA FOR FIRST TIME-STEP (LT=1)
 *         42   0.000   1.000

11.        NA   ALPH1      DA    ANGLE DATA FOR SECOND TIME-STEP (LT=8)
 *         16   0.000   1.000

COL 12345678901234567890123456789012345678901234567890123456789012345678901234567890
            1         2         3         4         5         6         7         8
```

DATA DECK FOR EXAMPLE 2.

LINES MARKED WITH * FORM THE ACTUAL DATA DECK. ALL OTHERS ARE COMMENTS ONLY.
NUMBERS ON LEFT CORRESPOND WITH SECTION NUMBERS IN SUMMARY.

```
COL          1         2         3         4         5         6         7         8
    12345678901234567890123456789012345678901234567890123456789012345678901234567890

 * 'SC4060 DATA CARD ACCORDING TO LOCAL INSTRUCTIONS'

 1. JOBNO  NTF  NTL  NDT   NF   NH   KA   KP   KG   KX   JP   JX
 *   297    1    1    1    3    1    3    3    2    6    1    1

 2.   MPT     MPRNT      MXPG     MXDAT
         MPLT      MPUN      MSIG
 *     2    2    1    1    1    1    1

 3.    NC       NIC     IC(2)
         NFMI     IC(1)     IC(3)
 *    18   80    3  401    6    2

 4. ARRAY C, ELEMENTS 1 TO NC
    IONO-CENTRE-COORDS  BASE-RAD  H-OF-BASE  H-D-TOP  NE-D-TOP   H-E-MAX  NE-E-MAX
 *  .0000E+00 .0000E+00 6430.E+00 60.00E+00 85.00E+00 2.500E+09 110.0E+00 1.000E+11
     H-F-MAX  NE-F-MAX    -NIGHT/DAY-NE-      --NIGHT-TO-DAY-TRANSITION--  H-ES-MAX
                          BASE     F-MAX     CENTRE  HALF-WIDTH SHIFT/TS
 *  300.0E+00 1.000E+12 .0000E+00 .3000E+00-1000.E+00 1000.E+00 500.0E+00 100.0E+00
     ----ES-LAYER------
     NE-MAX  HALF-WIDTH
 *  3.000E+11 1.000E+00

 5. ARRAY FMI - FORMAT TO PRINT IC AND C
 * (20H1    IONOSPHERE DATA/5X15H---------------//5X5HMODEL,I4,I8,11H BOUNDARIES//5
 * X15HTRANSITION TYPE,I2,33H (1 DAY-TO-NIGHT, 2 NIGHT-TO-DAY)///5X13HIONO C OF C (
 * ,F3.1,1H,,F3.1,16H)    BASE RADIUS,F7.1,2HKM///5X11HDAY PROFILE,7X6HHEIGHT,5X7HD
 * ENSITY//F29.1,4X9H.0000E+00/9X7HD-LAYER/F29.1,E13.4/9X7HE-LAYER/F29.1,E13.4/9X7H
 * F-LAYER/F29.1,E13.4///5X33HNIGHT/DAY DENSITY RATIO - AT BASE,F3.1,10H, AT F-MAX,
 * F3.1///5X42HTRANSITION REGION - RANGE TO CENTRE (LT=1),F8.1,2HKM/37X10HHALF WIDT
 * H,F8.1,2HKM/32X15HSHIFT/TIME-STEP,F8.1,2HKM///5X32HSPORADIC E-LAYER - HEIGHT OF
 * MAX,F7.1,2HKM/26X11HMAX DENSITY,E9.3/27X10HHALF WIDTH,F7.1,2HKM////)

 6. ARRAY ST - STEP LENGTHS IN EACH LAYER
      IN-D      IN-E      IN-ES     IN-E      IN-F
 *  10.000000  5.000000  0.100000  5.000000 10.000000

 7.     HM       RHOM       HEF
 *  299.0E+00 6000.E+00 100.0E+00

 8. ARRAY FREQ - SIGNAL FREQUENCIES
 *  16.000000 17.000000 18.000000

11.       NA   ALPH1      DA    ANGLE DATA FOR FIRST FREQUENCY (16MHZ)
 *        61   0.000   0.500

11.       NA   ALPH1      DA    ANGLE DATA FOR SECOND FREQUENCY (17MHZ)
 *        56   0.000   0.500

11.       NA   ALPH1      DA    ANGLE DATA FOR THIRD FREQUENCY (18MHZ)
 *        51   0.000   0.500

COL 12345678901234567890123456789012345678901234567890123456789012345678901234567890
            1         2         3         4         5         6         7         8
```

(iv) Calculation of signal strength (MSIG = 1). Mode split height (between E_S and F) is 100 km (HEF). Note: The change from E to E_S is smooth, so rays reflected in these layers are assumed to be in one mode in this context.

(v) Punched cards containing excess phase and group data and signal strength (MXDAT = 1). Again, since the rays are single hop, the value of JX (1 or 2) does not affect the punching order.

No other outputs (MPT, MPLT both 2). Note, therefore, that there are no input data cards corresponding to parts 9 and 10 of the input data summary.

```
IONOSPHERE MODEL USED IN EXAMPLE 2.
***********************************

     IONOSPHERE DATA
     ---------------

     MODEL 401        6 BOUNDARIES

     TRANSITION TYPE 2 (1 DAY-TO-NIGHT, 2 NIGHT-TO-DAY)


     IONO C OF C ( .0, .0)      BASE RADIUS 6430.0KM


     DAY PROFILE        HEIGHT     DENSITY

                         60.0     .0000E+00
         D-LAYER
                         85.0     .2500E+10
         E-LAYER
                        110.0     .1000E+12
         F-LAYER
                        300.0     .1000E+13


     NIGHT/DAY DENSITY RATIO - AT BASE  .0, AT F-MAX .3


     TRANSITION REGION - RANGE TO CENTRE (LT=1) -1000.0KM
                                      HALF WIDTH  1000.0KM
                                SHIFT/TIME-STEP    500.0KM


     SPORADIC E-LAYER - HEIGHT OF MAX  100.0KM
                          MAX DENSITY .300E+12
                           HALF WIDTH    1.0KM
```

9. ACKNOWLEDGMENTS

The author would like to express his thanks to Mr. H.H. Inston for helpful discussions during the development of the program and for his keen interest in its application to the problems studied by his team; to Mr. R.M. Jeffs for the original versions of the plotting subprograms; to Mr. K.B. Burchell for work on extending the outputs available from the program; and to Mr. T.L. van Raalte for advice on programming in general.

8. SUMMARY OF PROGRAM OUTPUTS

As can be seen in the listings, a variety of outputs are available from the program in the form of printed tables, punched cards and plotted ray-paths, and those required for a particular job are selected by a series of options entered as input data. A brief summary of the subroutines causing output and some cautionary notes on their use are given below. The descriptions and listings of each subroutine should be consulted for more detailed information.

(a) Subroutine PRNTPT gives a tabulation of five items of data for every ray-path point computed during a run. It is mainly intended for testing purposes and its use is not recommended for production work, as a very large quantity of printout is generated. Eg, during a typical short run (less than 5 min), about 20000 ray-path points may be computed. With the data for a maximum of 80 points being printed on one page, use of this print option would result in more than 250 pages of output.

(b) Subroutine RYPRNT produces a table summarising the key data for each ray-hop computed, and is a more practical form of printout than (a).

(c) Subroutine RYPNCH outputs punched cards containing the data of (b), 2 cards per ray-hop. This is a useful form of output when ray-data are required for other programs, eg, interpolation programs, but its indiscriminate use is not recommended due to the bulk of cards produced, eg, in the example of (a) above, about 200 typical ray-hops could be computed, resulting in an output of some 400 cards if this punch option is exercised.

(d) Subroutines EXPHGP and TŌTSIG produce printed tables summarising the results of some additional calculations.

(e) Subroutine DATAX causes the punching of cards containing the data produced by (d). The comment in (c) applies but there are only half as many cards. The data are also printed out, combining the tables of (d).

(f) Subroutine PLTRAY and its associated subroutines produce plots of the ray-paths on the SC4060 plotter unit. These can give a useful pictorial representation of the rays, but approximately double the length of time required for a run. A drawback that has become apparent since the subroutines were written is that in places where ray-path points are very close, eg, in a sporadic E-layer with step-lengths of the order of 0.1 km, the plots become fogged and unsuitable for reproduction, due to every point being plotted.

REFERENCES

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4. T.A. Croft: "The Synthesis of Oblique Ionograms by Digital Computer". Stanford Electronics Laboratories Technical Report No. 89 (September 1964)

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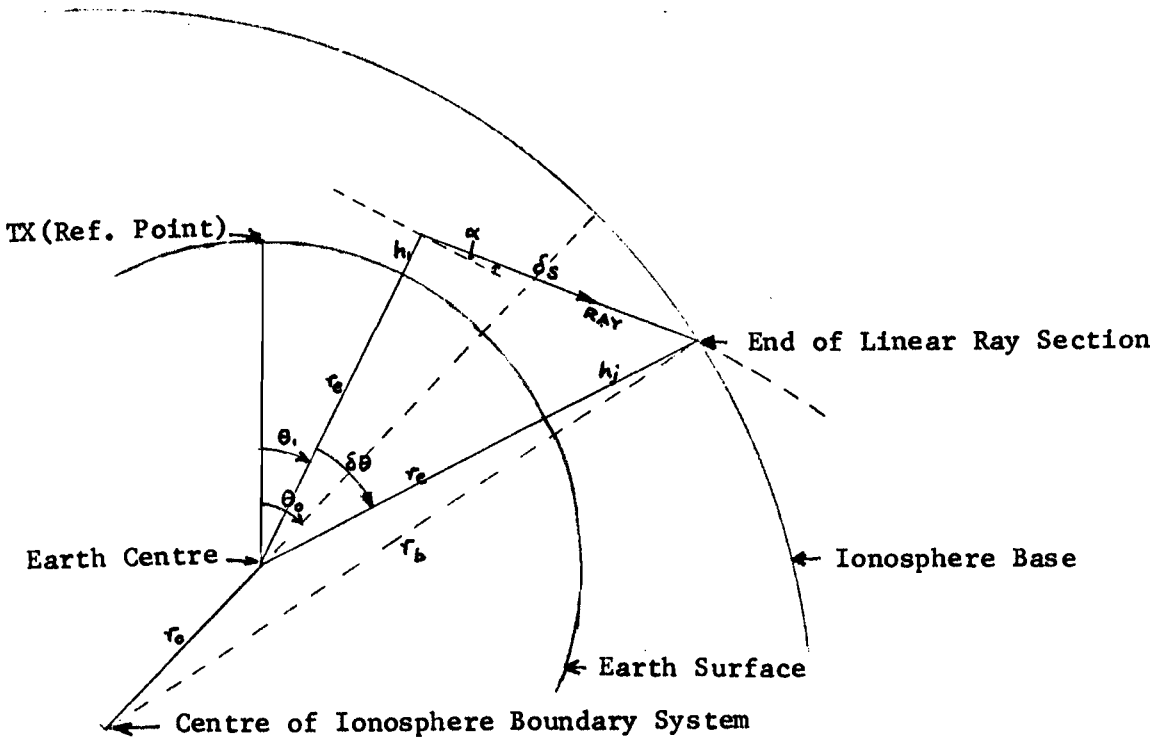

FIGURE 1. ASCENT OF RAY TO IONOSPHERE BASE

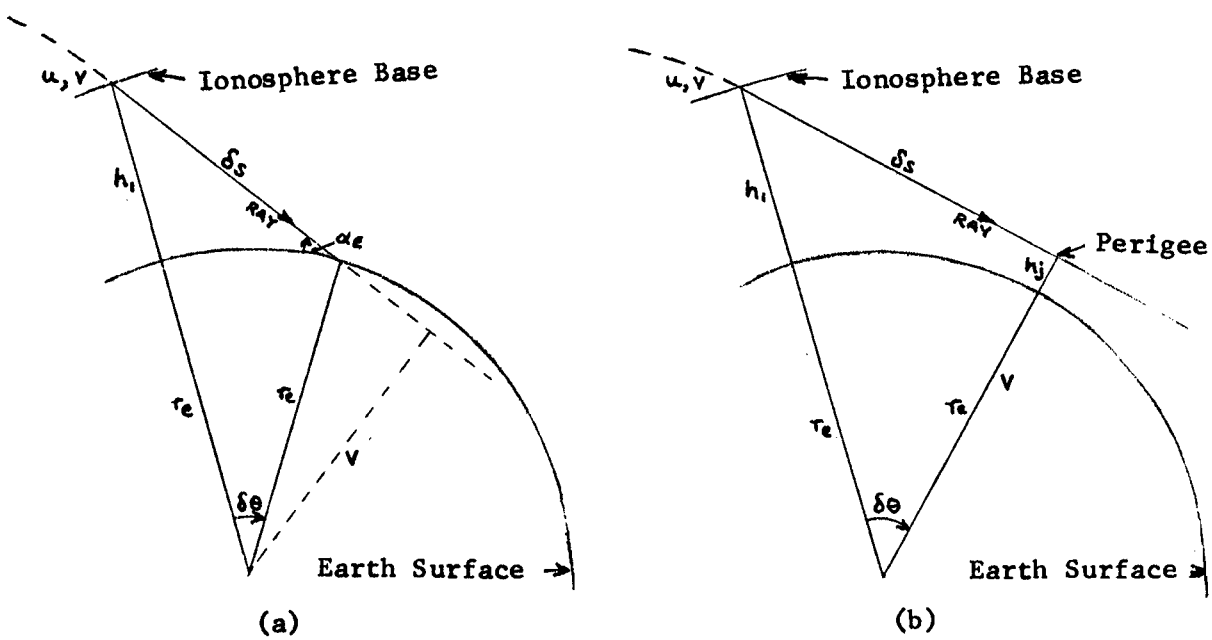

FIGURE 2. DESCENT OF RAY FROM IONOSPHERE BASE
(a) TO EARTH SURFACE
(b) TO PERIGEE

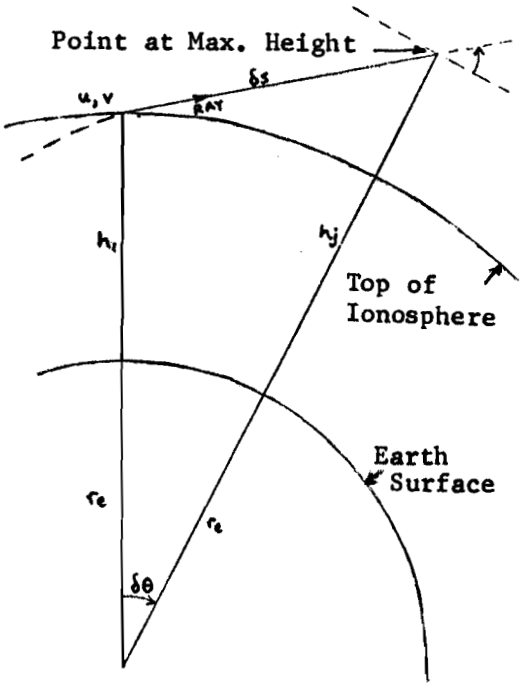

FIGURE 3. ASCENT OF RAY FROM TOP OF IONOSPHERE TO MAXIMUM HEIGHT

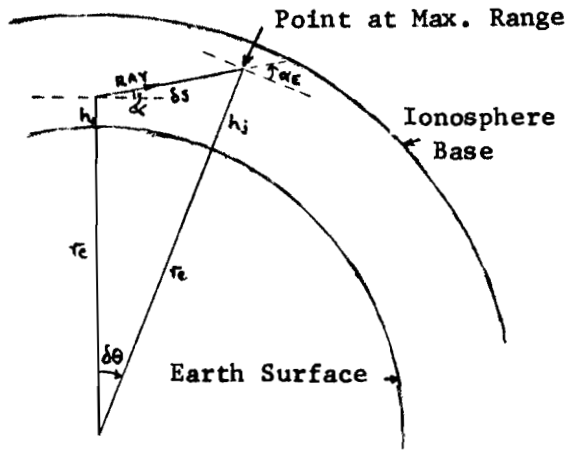

FIGURE 4. ASCENT OF RAY TO A POINT AT MAXIMUM RANGE BELOW IONOSPHERE

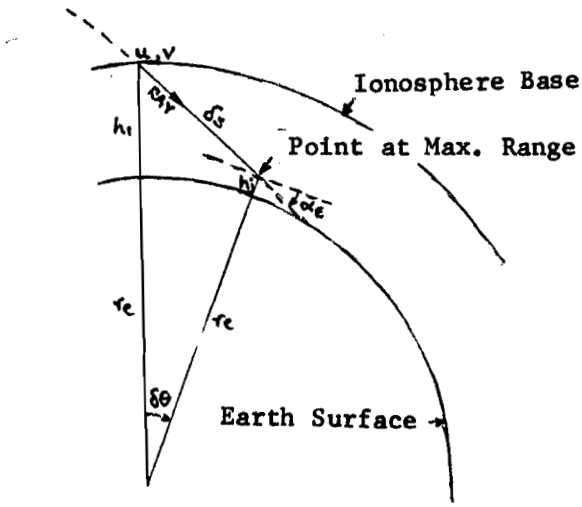

FIGURE 5. DESCENT OF RAY FROM BASE OF IONOSPHERE TO A POINT AT MAXIMUM RANGE

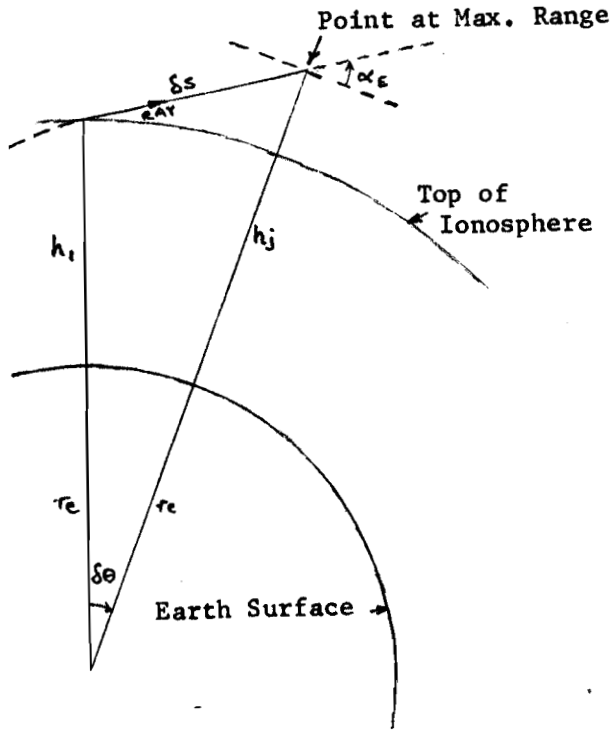

FIGURE 6. ASCENT OF RAY FROM TOP OF IONOSPHERE TO A POINT AT MAXIMUM RANGE

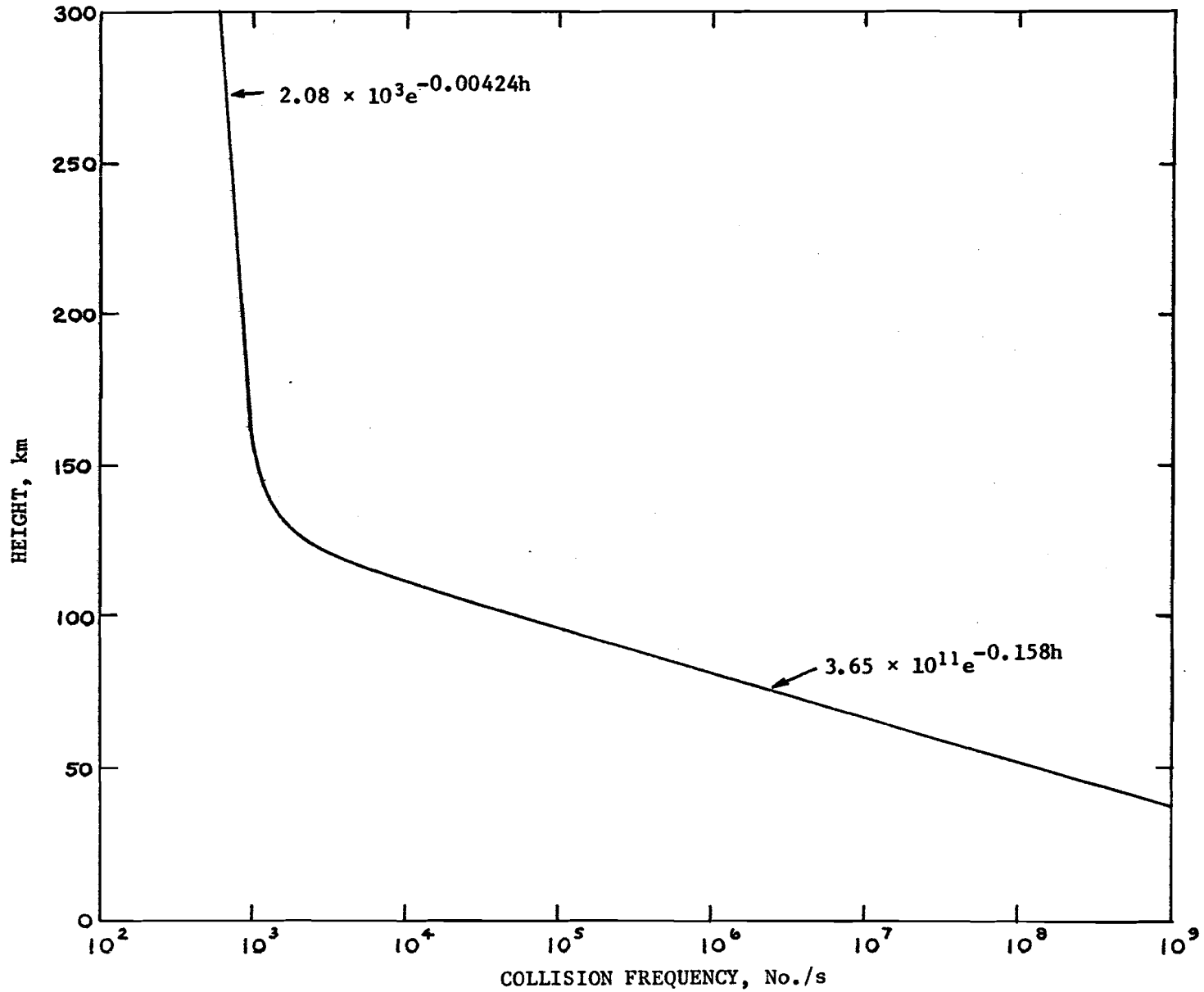

FIGURE 7. A MODEL OF COLLISION FREQUENCY

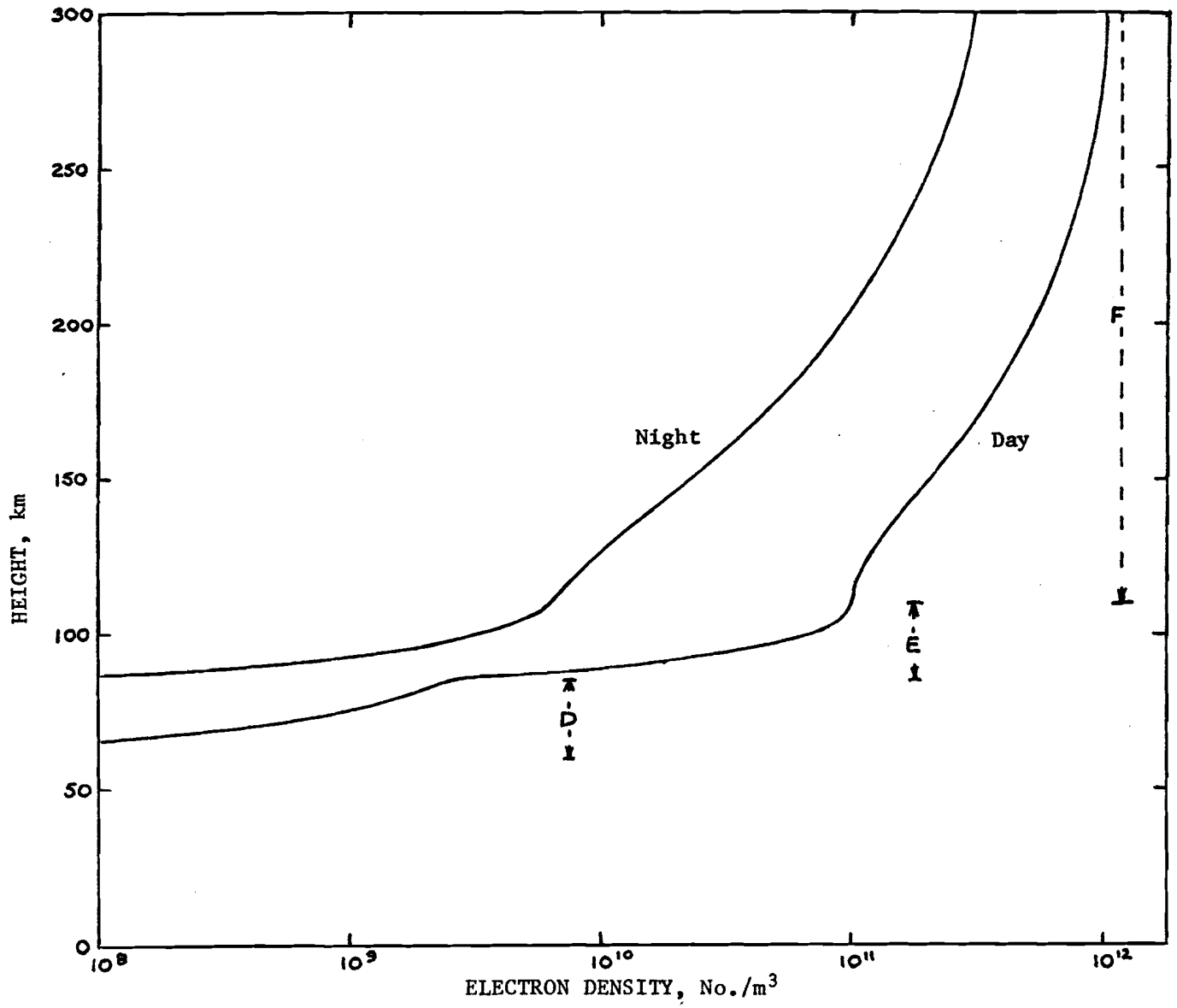

FIGURE 8. A 3-LAYER MODEL OF THE IONOSPHERE