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Global Seismicity in Terms of Short Period Magnitude m_b Based on Individual Station Magnitude-Frequency Distributions

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SUMMARY

Short period P wave amplitude data from selected arrays and from many stations contributing data to the ISC are analysed in terms of the Gutenburg-Richter magnitude frequency relation Log N(m) = a - bm. The data confirm the validity of the relation for global seismicity in the magnitude range $4.0 \leqslant m_b \leqslant 6.5$. Over 40 determinations of b, each based on over 1000 amplitude measurements indicate a mean global value of 0.98 ± 0.02 and using the observed numbers of moderate $5.75 < m_b < 6.05$ earthquakes to determine a, global seismicity is represented by the cumulative relation:-

$$\text{Log } N_c(m_b) = 7.77 - 0.98\ m_b.$$

This relationship indicates that for magnitude $m_b \geqslant 4.0$ the mean annual count is approximately 7900. This result is in good agreement with the average (7600) of 18 single station estimates of global seismicity.

1. INTRODUCTION

A knowledge of the number of earthquakes occurring both regionally and globally is an important consideration in the assessment of the resources required to monitor a Comprehensive Test Ban Treaty using seismological methods. Of particular importance is the number of earthquakes with magnitude (m_b) of 4.0 and greater. This magnitude is near the detection threshold of the most sensitive seismic stations operated to date and corresponds to explosions in granite with yields of 1 to 4 ktons.

An obvious way to estimate the number of earthquakes as a function of magnitude is to count those with magnitude and location determined by the International Seismological Centre (ISC), perhaps the most complete data source available. Figure 1 shows the global determinations of the ISC for the period 1964 to 1980 inclusive,plotted as a magnitude-frequency plot with intervals of 0.1 m_b units. In the magnitude range $5.3 \leqslant m_b \leqslant 6.2$ the data appear to follow a linear magnitude frequency law:-

$$\text{Log}_{10} N = a - bm \qquad \ldots (1)$$

where here

$$\text{Log}_{10} N = 9.81 - 1.44\ m_b \qquad \ldots (2)$$

with N equal to the number of events with magnitude $m_b \pm 0.05$.

Figure 1 also shows a similar plot for earthquakes within the "Soviet Bloc" (for definition see appendix A). Here the linear portion covers the range $5.0 \leqslant m_b \leqslant 6.2$ and has the form:-

$$\text{Log } N = 7.54 - 1.23\ m_b \qquad \ldots (3)$$

The "roll off" in numbers at low magnitudes is well known and results from the finite detection threshold for the world network submitting data to the ISC. Estimation of the numbers of earthquakes below m_b 5.0 therefore involves extrapolation of the linear relation (1). For the interval relations (2) and (3) the corresponding cumulative forms are:-

$$\text{Log } N_c = 10.29 - 1.44\ m_b \qquad \ldots (4)$$

$$\text{Log } N_c = 8.08 - 1.23\ m_b \qquad \ldots (5)$$

where N_c is the annual cumulative number with magnitude $\geqslant m_b$. If we extrapolate these curves down to $m_b = 3.95$ (corresponding to $m_b = 4.0$ as normally rounded) relations (4) and (5) give 40 000 and 1700 earthquakes annually for the world and Soviet Bloc respectively.

Systematic bias in the magnitudes obtained by simple network averaging as used by the ISC are now known to exist (eg, Ringdal (1,2), von Seggern and Blandford (3), von Seggern and Rivers (4)). This bias results from the truncation of the amplitude readings used to compute the mean (network) magnitude arising from non-detections at low magnitudes or saturated or clipped readings removed for high magnitudes. The overall result of this is a +ve bias at low magnitudes and a -ve bias at high magnitudes ($m_b > 6.0$) eg, von Seggern and Rivers (4). Unrealistically high activity in terms of the numbers of low magnitude earthquakes are thus obtained by downward extrapolation of equations (2-5).

Two different approaches can be used to solve the problem of network bias in the context of global seismicity estimation. A direct solution is to recompute the ISC magnitudes using a technique which allows for the truncation of data, as described by Ringdal (1). An alternative solution which circumvents the problem of network bias is to use amplitude data from individual stations to give "unbiased" estimates of a and b in equation (1). The remainder of this report concerns the latter but hopefully will complement the conclusions obtained from a recomputation of ISC magnitudes when they become available.

2. SEISMICITY ESTIMATION USING SINGLE STATION DATA

2.1 General considerations

Magnitudes determined by a single station can be used to construct magnitude frequency distributions and hence estimate a and b. Since the magnitudes will not be influenced by network averaging, the b value, clearly important if extrapolation is required, is unbiased (von Seggern and Blandford (3)). The "activity" constant a, requires correction however as demonstrated by von Seggern and Blandford (3). The correction is:-

$$\delta a = -b(1.15b\sigma_s^2 + C) \qquad \ldots (6)$$

where σ^2 is the variance of the station magnitude estimates about the true value. The factor $1.15b\sigma_s^2$ accounts for the net "spill over" from the more frequent smaller events towards larger magnitudes arising from σ_s. Typically σ_s = 0.3 to 0.4 m_b units, so for b = 1.0, a is reduced by about 0.15 and predicted seismicity by 30%. The station correction C accounts for the sub-station crustal response and attenuation. Typically C ranges between ±0.3 m_b units which would influence seismicity estimates by a factor of 2 either way. Finally a correction to a, not given in (6), must be applied to account for the fraction of the globe not effectively sampled by the station, ie, the core shadow. Clearly the effect of errors in a on seismicity counts using single stations can be large and therefore the results presented here concentrate on the accurate determination of b.

2.2 Advantages of the use of amplitudes over magnitudes

An important consideration in the determination of b is whether frequency distributions are constructed using the raw amplitude data or in terms of magnitudes m_b. Short period body wave magnitudes (m_b) are computed from the measured ground amplitude A (microns) and period T (s) using the relation:-

$$m_b = \mathrm{Log}_{10}\frac{A}{T} + Q(\Delta,h) \qquad \ldots (7)$$

where $Q(\Delta,h)$ is the Gutenburg and Richter (5) distance (Δ) and focal depth (h) correction factor. Clearly if the earthquakes come from a region of limited spatial extent $Q(\Delta,h)$ is constant and the observed magnitude-frequency and amplitude-frequency distribution are the same apart from a factor $Q(\Delta,h)$ subtracted from a. (Note: Here and elsewhere in this report the word amplitude has been used to denote $\mathrm{Log}_{10}\frac{A}{T}$). If the seismicity is from an extended region with $Q(\Delta,h)$ varying, provided b is constant, the use of amplitude instead of magnitude still conserves b in the frequency distribution, although the significance of a in terms of seismicity is obscured. The use of amplitudes instead of magnitudes in the determination of b values has two advantages. Since it is not necessary to compute $Q(\Delta,h)$ from the distance and depth of the earthquakes it is possible to use the full set of observations from sensitive stations, including data from sources too small to be located by the ISC. Variation of $Q(\Delta,h)$ also means that the detection threshold in terms of m_b at a station is region dependent. This variation inevitably broadens the "roll off" region of the frequency distributions and restricts or may even remove the linear section.

2.3 Advantage of joint estimation of seismicity and detection threshold

Even when amplitudes are used, determination of a and b alone involves the difficult and subjective decision concerning the linear range of the observed distribution. Truncation of the data below the assumed "100%" detection level also removes much useful data concerning the magnitude frequency low at small magnitudes. Kelly and Lacoss (6) have described joint estimation technique which avoids this by determining a and b together with parameters concerned with the detection threshold.

The method assumes a specific form for the frequency distribution of amplitudes near the threshold however, and may not be appropriate when the "roll off" is complicated by variation in $Q(\Delta,h)$ when magnitudes are used or where low amplitude readings are removed because of non-association with locatable events, as in data published by the ISC. Nevertheless some results obtained using joint estimation are presented below since they provide confirmation on the applicability of the frequency law over the widest range of amplitudes and are also a check on the results obtained using data sets such as that from the ISC where selection by association has been performed.

2.4 Joint estimation of detection threshold and seismicity

2.4.1 Introduction

A procedure described by Kelly and Lacoss (6) enables the joint estimation of the seismicity constants a and b together with the (50%) detection threshold μ and its standard deviation σ. A full description of this method and some modifications made for use here is given in appendix B. The data required are the complete set of amplitude observations whether associated with known (ie, located) sources or not. Three sources of data were chosen which in general detect many events not located by the ISC. These sources are:-

(1) The large aperture NORSAR array (NAO,NB2)

(2) The medium aperture arrays Warramunga (WRA), Australia, Gauribidanur (GBA), India and Eskdalemuir (EKA), Scotland.

(3) The small aperture VELA uniform arrays BMO, CPO, TFO, UBO and WMO.

2.4.2 NORSAR data

Amplitude data for the NORSAR array were obtained on magnetic tape. Prior to 1976 the array operated with a full aperture of 120 km (code NAO) but since that date it has operated with a reduced aperture of 60 km, (usually code NB2) and two representative sets of data have therefore been analysed. The amplitude frequency data and fitted curves are in figure 2. For both sets good fits indicate that both the magnitude frequency law and the form of the curve near the detection threshold are appropriate over a range of amplitudes from near 2.5 down to 0.0 or less (equivalent to m_b = 4.0 to 6.5 at teleseismic distances). For NAO no amplitude was reported if any of the array elements were overloaded which results in a rather sudden truncation of the data above amplitudes of 2.3. In contrast a single element low gain instrument is used to measure the larger amplitudes at NB2, so only the lower amplitude data have been subject to array summing. This may account for the somewhat irrregular variation about the linear section. Estimated b values are both low (0.83 to 0.88, see table 1) and for NAO is in good agreement to the value 0.83 found by Bungum and Husebye (7) based on 1 years data.

2.4.3 EKA, GBA, WRA arrays

Amplitude data for these arrays are for arrivals detected and measured by automatic processors (Key, Lea and Douglas (8)) installed at each station. As with NORSAR the fitted curves are a good approximation to the data apart from some data loss at high amplitudes caused by overloading at the more sensitive GBA and WRA arrays (figure 2). Taking the data as a whole, the amplitude-frequency law and the form of the threshold, suggested by Kelly and Lacoss (6) are confirmed for the amplitude range 3.0 (see EKA) to near 0.0 (see WRA). The b values, given in table 1, are again less than 1.0 except for EKA (1.15) which is based on a rather small sample of amplitudes.

2.4.4 VELA arrays

The VELA arrays operated from the early 1960's to the early 1970's and had a considerable impact on the seismic-event-detection capability of the world network. Arrival time readings, amplitude and period data are published in bulletins, where association of readings with known located earthquakes is made. Non-associated readings are classified as local, regional or teleseismic ($\Delta \geqslant 15.5^{\circ}$) on the basis of the characteristics of the observed waveform. Amplitude data for all teleseismic earthquakes, associated or not, were extracted from the published bulletins for the year 1966. This year is well into the full operational period of the arrays and is free from major aftershock-sequences, otherwise the choice is arbitrary. With the exception of BMO, joint determination of seismicity and threshold parameters gives unsatisfactory results (figure 3) the assumed form of the curves near the thresholds is clearly inappropriate. The excess numbers near the thresholds are presumably false identifications at or near the noise level. Revised b values in table 1 are based on fits to the linear sections alone which are typically from amplitudes in the range 1.0 to 2.5. The values range from 1.05 to 1.15 and compare with a value of 0.93 obtained by Chinnery (9) from ISC event assoicated readings for the period 1966 to 1970.

2.4.5 Discussion

The b values obtained from these arrays (table 1) suggest that the average global b value based on ISC mean magnitudes is biased 0.36 to 0.6 units high. Scatter in the b values (0.83 to 1.15) will be in part statistical (see appendix B) but some significant variation could be present. The cause of such variation is not investigated here but may be the result of subtle combinations of the spectral-magnitude-scaling law for the earthquakes with either the instrument response or substation structural response, or with arrays could result from beamforming loss. These data confirm that the amplitude-frequency distribution in linear over a wide range (0.0 to 2.5) which for the predominently teleseismic arrivals correspond to magnitudes in the range $4.0 \leqslant m_b \leqslant 6.5$. Alternative b values computed using the assumed linear section are (apart from the VELA arrays) in good agreement with the joint analyses and have a mean value of 1.05.

2.5 Estimation of single station b values using ISC data

2.5.1 Introduction

The variation in b values found in the previous section has fairly wide limits and it is clearly desirable to use data from a larger suite of stations from other regions. Amplitude data obtainable on ISC bulletin tapes have proved to be a useful source of such data. To be useful stations must be sensitive enough and/or have submitted data for long enough to give sufficient amplitudes to enable stable b value estimates. In addition it is important that the threshold for the measurement and submission of amplitude should be as constant as possible since any variation obscures the start of the linear part of the frequency distributions. The removal of amplitude readings for non-associated earthquakes may distort the frequency distribution near the threshold, especially for the most sensitive stations. Joint analysis of the frequency distribution as described in the previous section was therefore not undertaken, instead, it was replaced by the simplified fit to the linear region even though this invokes the decision concerning the linear range discussed above.

2.5.2 Selection of data

ISC data for the period 1970 to 1981 inclusive for a large number of stations were first examined on a monthly basis in terms of the total number and minimum of the amplitudes reported. Only data in the distance range 21 to 100° were considered, so as to be consistent with the range used in the ISC's own magnitude estimates. This examination isolated the time periods in which the ISC was regularly receiving data and any substantial periods of station "down-time". Changes, both short and long term, in the monthly counts frequently correlated with the minimum amplitudes and suggest variation in sensitivity and station procedures. Data from within time periods showing large variation in effective station performance were deleted from further analysis. Finally, only stations which had at least 250 readings on the observed linear part of the frequency distribution were used which ideally should give b determinations accurate to ±0.2 or better at the 95% level (see appendix C).

2.5.3 Results

The locations of the earthquakes have been determined for the ISC data and so analysis in terms of magnitude rather than amplitude is possible with the estimates of a, interpretable in terms of seismicity. Variation of the distance factor $Q(\Delta,h)$ on the range 20 to 100° is considerable however and so analysis was again made in terms of amplitudes in order to optimise the determination of b.

Altogether data from 88 stations were processed using methods suggested by Page (10) and Bender (11) to determine the constants a and b. The full results are shown in table 2. As expected (see appendix C) results for the two methods agreed. To illustrate the quality of the linear fits the frequency distributions for those stations contributing over 1000 amplitudes to the b value determinations are reproduced in figure 4. The lower limit on the amplitude range used to determine a and b was chosen carefully to be on the linear section. Exact choice of the upper limit was found to be less critical using the maximum likelihood method but was made high enough to give a total amplitude range of 1.0 unit or more. Assuming no systematic errors, confidence limits for these determinations should all be less than ±0.1 at the 95% level (appendix C).

The frequency distribution of the full 88 determinations of b is shown in figure 5 and also for the subset containing only those based on more than 1000 amplitude readings. Both sets have a modal value of b = 1.0. The full set has a mean of 1.06, while for the subset the mean is 0.98. A clear skewness in the full distribution is not present in the subset because of the elimination of the determinations mainly from two networks of stations in France and French Polynesia which represent some 20% of the total. Without more detailed information with respect to the instrument responses, overload levels and procedure for selecting data sent to the ISC, the reason for these high b values for French stations remains obscure. It is clear however, that these data have a disproportionate weight and therefore the subset containing the more accurately determined b values is regarded as more representative globally. The standard deviation of the b value distribution of the latter is ± 0.10, twice that which might be expected from the statistical variation of the estimates (appendix C). The confidence limits are however based on idealised data and scatter using real data will inevitably be greater. The data does not permit the demonstration of any convincing correlations of b either with station correction or region.

3. WORLD AND SOVIET SEISMICITY ESTIMATES

3.1 Extrapolation of ISC magnitude statistics

Results described above suggest that global seismicity in terms of m_b can be described by a linear-magnitude-frequency law (1) in the approximate range $4.0 \leqslant m_b \leqslant 6.5$ with a mean b value of 0.98 ± 0.02. Global estimates of earthquake numbers as a function of m_b can therefore be found provided we have an unbiased estimate of the activity constant a. Such an estimate can be obtained from ISC magnitudes in the range where network truncation effects lead to minimum bias in computed magnitudes.

The bias introduced to magnitude determinations resulting from truncation of data at noisy stations must decrease with increasing m_b. For the ISC data the majority (but not all) of stations appear to have near 100% detection above amplitudes (Log $\frac{A}{T}$) of 1.5 (see table 2 giving lower limit of ranges used for b value estimation). Assuming the Gutenburg-Richter (1956) distance factors for shallow events, this is equivalent to m_b values of 4.6 to 6.0 in the distance range 21 to 100° as used by the ISC. Unfortunately above magnitude (m_b) 6.1, the network bias resulting from the problem of overloaded records becomes manifest (see figure 1 and also von Seggern and Rivers (4)). A compromise solution is to use earthquake counts in the range 5.8 to 6.0 inclusive (in reality 5.75 to 6.05) which will have the minimum bias and yet have enough events distributed globally to give a reasonably accurate value for a.

For the 17 years 1964 to 1981 inclusive the ISC reported 1147 earthquakes with magnitudes between 5.75 and 6.05. Assuming b = 0.98, a representative figure considering the observed distribution and scatter described in the previous section, the global annual seismicity can be described by:-

$$\mathrm{Log}_{10} N_C = 7.77 - 0.98\, m_b \qquad \ldots (8)$$

where N_C is the cumulative number of earthquakes with magnitude $\geqslant m_b$ annually. This relation gives a total of approximately 7900 earthquakes with $m_b \geqslant 4.0$ (3.95) annually.

It is also possible, assuming the b value is appropriate regionally, to use the ISC activity between m_b 5.75 to 6.05 to determine regional values for a. For the Soviet Bloc (as defined in appendix A) there were 103 such earthquakes in the 17 years and this gives the cumulative distribution:-

$$\mathrm{Log}_{10} N_C = 6.73 - 0.98\, m_b \qquad \ldots (9)$$

indicating approximately 720 earthquakes annually. This is dominated by activity in the Kamchatka-Kuriles region with most activity located beneath the ocean. Excluding this region there were only 31 earthquakes in the desired magnitude range and so for the main Soviet Bloc land mass we have:-

$$\mathrm{Log}_{10} N_C = 6.21 - 0.98\, m_b \qquad \ldots (10)$$

resulting in approximately 210 $m_b \geqslant 4.0$ earthquakes annually. Table 3 summarises the seismicity statistics outlined in this section.

The accuracy of these estimates depends on the assumption that the ISC magnitudes in the range 5.8 to 6.0 are unbiased, the validity of the value 0.98 for b and any statistical variation on the seismicity counts used to determine a. The latter can become significant for regions of relatively low activity (see table 3) but for the global seismicity the effect of an error in b is the dominant factor. Assuming the mean b of 0.98 is unbiased with standard error ± 0.02 the corresponding range in the global $m_b \geqslant 4.0$ seismicity is 7400 to 8500.

3.2 Individual station estimates of global seismicity

A check on the predictions of equation 8, totally independent of ISC magnitudes, is obtainable from single-station-magnitude frequency distributions (which use m_b rather than Log $\frac{A}{T_o}$) as outlined in section 2.1. If only data from the teleseismic range Δ = 30 to 90° are used the deleterious effect of variation in the distance correction $Q(\Delta,h)$ on the identification of the linear section of the distributions is minimal. This range restriction however, results in a drop in the number of contributing amplitudes and the analysis was restricted to 18 stations, with over 1000 readings remaining, mainly in Europe and North America. Results are given in table 4. The a values, which are given for the cumulative counts are corrected according to equation (6) and also to account for the percentage of world seismicity in the distance range (30 to 90°) used. The full correction to a is then:-

$$a_c = a - b(1.15b\sigma^2 + C) - \mathrm{Log}_{10} F_T \qquad \ldots (11)$$

where the term in brackets is equation (6) and F_T is the fraction of global seismicity in the teleseismic range. Values of F_T in table 4 are derived from the fraction of all large ($5.8 \leqslant m_b \leqslant 6.0$) earthquakes observed by the ISC in the distance range $\Delta = 30$ to 90°. Station corrections C and magnitude variance σ^2 are taken from North (12). In the last column ($N\omega$) gives the estimates of global $m_b \geqslant 4.0$ (3.95) seismicity from the individual station a_c and b values. The values have a mean of 7600 ± 700 and a standard deviation of 3000. True numbers must be somewhat higher because no corrections are available for station "down-time". This result is in reasonable agreement with that of the previous section concerning global earthquake numbers.

4. CONCLUSIONS

Global seismicity measured on the short-period body-wave scale can be represented by the cumulative distribution:-

$$\mathrm{Log}_{10} N_C = 7.77 - 0.98\, m_b \qquad \ldots (12)$$

and is applicable in the range $4.0 \leqslant m_b \leqslant 6.5$ at least. Extrapolation of the distribution indicates approximately 7900 $m_b \geqslant 4.0$ (3.95) earthquakes annually. Individual station estimates of seismicity confirm this figure. For the Soviet Union including the Kurile-Kamchatka region some 720 such earthquakes occur annually. Excluding the active Kurile-Kamchatka region however, the number of events with $m_b \geqslant 4.0$ is approximately 210.

5. ACKNOWLEDGMENTS

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APPENDIX A

DEFINITION OF SOVIET BLOC

The Soviet Bloc here was defined by the Flinn and Engdahl (13), region numbers which include the Soviet Union and border regions and also regions normally regarded as in the Soviet "sphere of influence" (ie, Eastern Europe). East Germany is not included as it has not been separately regionalised and does not include any large earthquakes.

The region numbers are:-

4, 217, 218, 219, 220, 221, 320, 326, 327, 328, 329, 330, 331, 333,

334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 349, 350, 357, 358,

359, 360, 361, 362, 363, 367, 383, 391, 392, 547, 548, 549, 645,

648, 649, 650, 653, 656, 657, 661, 662, 663, 668, 670, 671, 709,

713, 714, 715, 716, 717, 718, 719, 721, 722, 723, 724, 725, 726.

The regions are delineated in figure 6.

APPENDIX B

COMPUTATION OF SEISMICITY AND THRESHOLD PARAMETERS

A2.1 Introduction

Three methods for estimating the seismicity parameters a and b in equation (1) and the station detection thresholds are used in this report. These have been published by Page (10), Kelly and Lacoss (6) and Bender (11). The latter assumes that the magnitude data are grouped into discrete intervals and a full description is given in the paper. The Page (10) and Kelly and Lacoss (6) methods assume a continuous magnitude distribution. Some changes in the latter are required to take account of a possible upper limit on observed magnitudes not considered in the original theory. This appendix outlines the theory of the Kelly and Lacoss (6) method incorporating these modifications. The Page method is derived as a special case of this theory.

A2.2 General forms of the likelihood equation for continuous magnitudes

Assume that the observed magnitudes are from a population following the standard magnitude frequency law:-

$$N(m) = 10^{(a-bm)} \qquad \ldots (13)$$

or

$$N(m) = e^{(\alpha-\beta m)} \qquad \ldots (14)$$

where

$$\alpha = a \, \mathrm{Log}_e(10)$$

$$\beta = b \, \mathrm{Log}_e(10).$$

N(m) = number of earthquakes per annum with magnitude m (per unit magnitude).

Let the probability that an earthquake of magnitude m is detected and magnitude measured be described by a function D(m) of m. In general D(m) will involve further parameters yet to be specified. From (14) the number of observed magnitudes is therefore:-

$$N(m) = D(m)e^{(\alpha-\beta m)}. \qquad \ldots (15)$$

If the observed magnitudes have an upper limit M_{max} then the average total number of observed magnitudes $\bar{N}$ is given by:-

$$\bar{N} = \int_{-\infty}^{M_{max}} D(m)e^{(\alpha-\beta m)}dm. \qquad \ldots (16)$$

The probability that an observed magnitude has a value m is given by the probability density function f(m) which from (15) and (16) is:-

$$f(m) = \frac{D(m)e^{(\alpha-\beta m)}}{\bar{N}} . \qquad \ldots (17)$$

Suppose the total observed number of magnitudes is K each with magnitude m_i. If K is poisson distributed with rate $\bar{N}$ then the probability of observing exactly K magnitudes is:-

$$p(K) = \frac{\bar{N}^K e^{-\bar{N}}}{K!} . \qquad \ldots (18)$$

Hence the overall probability $P(K,m_1 \ldots m_K)$ of observing K magnitudes with magnitudes m_i is the product of probabilities given by (17) and (18)

ie,

$$p(K,m_1 \ldots . m_K) = p(K) \prod_{i=1}^{i=K} f(m_i). \qquad \ldots (19)$$

Substituting (17) and (18) into (19)

$$P(K,m_1 \ldots . m_K) = \frac{e^{-\bar{N}}}{K!} \prod_{i=1}^{i=K} D(m_i)e^{(\alpha-\beta m_i)} \qquad \ldots (20)$$

Taking logs of both sides we get the likelihood equation

$$L = -\bar{N}-\mathrm{Log}K! + K\alpha - \beta \sum_{i=1}^{i=K} m_i + \sum_{i=1}^{i=K} \mathrm{Log}(D(m_i)) \qquad \ldots (21)$$

where L is the likelihood.

Maximum likelihood estimates can be found by maximising L for α,β and any parameters not yet specified in D(m).

For a turning point

$$\frac{\partial L}{\partial \alpha} = 0,$$

Hence from (21)

$$-\frac{\partial \bar{N}}{\partial \alpha} + K = 0, \qquad \ldots (22)$$

but from (16)

$$\frac{\partial \bar{N}}{\partial \alpha} = \bar{N}, \qquad \ldots (23)$$

Hence (22) gives

$$K = \bar{N}. \qquad \ldots (24)$$

The maximum likelihood solution is obtained when the predicted count $\bar{N}$ is set equal to the observed count K (a well known result irrespective of the form of D(m)).

Substituting (24) into (16) and rearranging:-

$$\alpha = \mathrm{Log}K - \mathrm{Log}\left(\int_{-\infty}^{M_{max}} D(m)^{(-\beta m)} dm \right). \qquad \ldots (25)$$

From (24) and (25) we can now eliminate from the likelihood equation (21),

$$L = -K - \mathrm{Log}K! + K\mathrm{Log}K - K\mathrm{Log}\left(\int_{\infty}^{M_{max}} D(m)^{-\beta m} dm \right) - \beta \sum_{i=1}^{i=K} m_i + \sum_{i=1}^{i=K} \mathrm{Log}(D(m_i)). \qquad \ldots (26)$$

To proceed further it is now necessary to specify the form of D(m).

A2.3 Derivation of Page formula

Determination of β from the observed linear part of the magnutide frequency is equivalent to assuming D(m) has the simple form:-

$$D(m) = 1.0 \text{ for } M_{min} \leq m \leq M_{max} \text{ (total detection)} \qquad \ldots (27)$$

$$D(m) = 0.0 \text{ elsewhere (ie, no data used)}$$

The likelihood (26) is now a function of β only.

$$L = -K - \mathrm{Log}K! + K\mathrm{Log}K - K\mathrm{Log}\left(\int_{M_{min}}^{M_{max}} e^{-\beta m} dm \right) - \beta \sum_{i=1}^{i=K} m_i \qquad \ldots (28)$$

$$L = -K - \mathrm{Log}K! + K\mathrm{Log}K - K\mathrm{Log}\left(\frac{e^{-\beta M_{min}} - e^{-\beta M_{max}}}{\beta} \right) - \beta \sum_{i=1}^{i=K} m_i \qquad \ldots (29)$$

At turning points

$$\frac{\partial L}{\partial \beta} = 0.$$

Therefore from (29)

$$\frac{-K\,(M_{max}e^{-\beta M_{max}} - M_{min}e^{-\beta M_{min}})}{e^{-\beta M_{min}} - e^{-\beta M_{max}}} + \frac{K}{\beta} - \sum_{i=1}^{i=K} m_i = 0, \qquad \ldots (30)$$

hence

$$\frac{1}{\beta} = \langle m \rangle - \left(\frac{M_{min} - M_{max}\, e^{-\beta(M_{max} - M_{min})}}{1 - e^{-\beta(M_{max} - M_{min})}} \right) \qquad \ldots (31)$$

where $\langle m \rangle$ is the mean magnitude ($M_{min} \leqslant m \leqslant M_{max}$).

Equation (31) is the Page (10) formula and can be solved numerically for β given observed $\langle m \rangle$, M_{max} and M_{min}. α can then be obtained from (25) with the assumed form of D(m) (27).

$$\alpha = \mathrm{Log}K - \mathrm{Log}\left(\frac{e^{-\beta M_{min}} - e^{-\beta M_{max}}}{\beta} \right) \qquad \ldots (32)$$

A2.4 <u>Inclusion of detection threshold parameters</u>

The form of the detection probability D(m) used by Kelly and Lacosse is:-

$$D(m,\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{m-\mu} e^{\frac{-y^2}{2\sigma^2}}\, dy. \qquad \ldots (33)$$

The probability is therefore near zero for small m, equals 0.5 for $m=\mu$ and tends to unity for large m. Hence the detection capability is determined by the 50% threshold μ and the variance σ^2.

Substituting (33) into the likelihood equation (26):-

$$L = -K - \mathrm{Log}K! + K\mathrm{Log}K - K\mathrm{Log}\left(\int_{-\infty}^{M_{max}} D(m,\mu,\sigma)e^{-\beta m} dm \right) - \beta \sum_{i=1}^{i=K} m_i + \sum_{i=1}^{i=K} \mathrm{Log}(D(m_i,\mu\sigma)), \qquad \ldots (34)$$

at turning points

$$\frac{\partial L}{\partial \beta} = 0$$

hence

$$\frac{\int_{-\infty}^{M_{max}} D(m,\mu,\sigma) m e^{-\beta m} dm}{\int_{-\infty}^{M_{max}} D(m,\mu,\sigma) e^{-\beta m} dm} - \langle m \rangle = 0, \qquad \ldots (35)$$

where

$$\langle m \rangle = \frac{1}{K} \sum_{i=1}^{i=K} m_i \text{ the mean magnitude,}$$

Similarly two further equations can be obtained by differentiating with respect to μ and σ but since the equations become intractable the following numerical procedure has been found sufficient. Starting with approximations to μ and σ compute β by solving equation (35) numerically. Substitute β into (34) and maximise the likelihood numerically for variation of μ and σ. Using new μ and σ recompute β and iterate. After convergence α is obtained from (25). It is worth noting that equations (25), (34), and (35) are equivalent to (38), (39) and (45a) in Kelly and Lacoss (6) where the assumption of infinite M_{max} enables direct evaluation of the integrals.

APPENDIX C

CONFIDENCE LIMITS ON b VALUE ESTIMATES

The problem of assigning confidence limits on estimated b values is discussed elsewhere in the literature (eg, Aki (14), Utsu (15), Weichert (16)). In a more recent paper Bender (11), has investigated both confidence limits and the problem of bias introduced on b values computed using a range of commonly used methods. Following Bender (11), tests using simulated data are used here to check the techniques employed for any bias present and also obtain the distribution of estimated b values.

Two maximum likelihood formulae were used to estimate b in this report from the linear sections of the observed magnitude frequency distributions. The Page (10) formula described in appendix B assumes that the data is from a continuous distribution within a magnitude range M_{max} to M_{min}. The other formula used is described fully by Bender (11) and assumes that the data from the continuous parent population has been grouped into discrete magnitude intervals. For the ISC data where amplitudes and periods are either given separately or more frequently as $Log(\frac{A}{T})$ rounded to the nearest 0.1 unit, either method may be appropriate for use on the resulting magnitude data.

To test both methods simulated sets of data grouped at 0.1 unit magnitude intervals were generated from a parent population following the continuous magnitude frequency distribution with b=1.0. The range of magnitudes M_{max} -M_{min} was assumed to be either 1.1 and 2.5 representing the observed upper and lower limits for real data. Sets of 1000 simulations were made for sample sizes varying from 80 to 5000 and from the resulting distribution of b values, the mean, median and 95% confidence bounds on b computed.

For both methods the resulting mean and median b values differed from 1.0 by less than 1% indicating that any bias present in the results introduced by the analysis method can be neglected. Figure 7 shows the 95% confidence bounds as a function of sample size for data from the limited (M_{max}-M_{min} = 1.1) magnitude range and also the maximum (M_{max}-M_{min} = 2.5) likely to be observed. It is clear that large sample sizes are required for accurate b value determinations. At least 250 observations are required to achieve 0.2 accuracy at the 95% level 800 or more observations are needed to achieve +0.1. Simulations assuming a population b value of 0.75 and 1.25 give similar results. As indicated by Bender (11) the error limits for the same sample size are reduced if the data is from a wider range of magnitudes and for the range M_{max}-M_{min} = 2.5 the results (figure 7) approximate the formula proposed by Aki (14),

$$\sigma_b = b/\sqrt{N}$$

for the standard deviation of estimates of b from a sample of size N with unlimited upper magnitude (ie, M_{max}-M_{min} = $\propto$).

REFERENCES

1. F Ringdal: "Maximum-Likelihood Estimation of Seismic Magnitude". Bull Seism Soc Am, 66, 789-802 (1976)

2. F Ringdal: A Reply to "Comments on the Use of Truncated Distribution Theory for Improved Magnitude Estimation" by D von Seggern and D W Rivers. Bull Seism Soc Am, 68, 1547-1548 (1978)

3. D von Seggern, R Blandford: "Seismic Threshold Determination". Bull Seism Soc Am, 66, 753-788 (1976)

4. D von Seggern, D W Rivers: "Comments on the Used Truncated Distribution Theory for Improved Magnitude Estimation". Bull Seism Soc Am, 68, 1543-1546 (1978)

5. B Gutenburg, C F Richter: "Magnitude and Energy of Earthquakes". Anali Geofis, 9.1 (1956)

6. E J Kelly, R T Lacoss: "Estimation of Seismicity and Network Detection Capability". MIT Lincoln Lab, Tech Note 41 (1969)

7. H Bungum, E S Husebye: "Analysis of the Operational Capabilities for Detection and Location of Seismic Events at NORSAR". Bull Seism Soc Am 64, 637-656 (1974)

8. F A Key, T G Lea and A Douglas: "Seismometer Array Station Processors". AWRE Report 0-36/76, HMSO (1977)

9. M A Chinnery: "Measurement of m_b with a Global Network". Tectonophysics, 49, 139-144 (1978)

10. R Page: "Aftershocks and Microaftershocks of the Great Alaska Earthquake of 1964". Bull Seism Soc Am, 58, 1131-1168 (1968)

11. B Bender: "Maximum Likelihood Estimation of b Values for Magnitude Grouped Data". Bull Seism Soc Am, 73, 831-851 (1983)

12. R G North: "Station Magnitude Bias - Its Determination, Causes and Effects". MIT Lincoln Lab, Tech Note 24 (1977)

13. E A Flinn, E R Engdahl: "A Proposed Basis for Geographical and Seismic Regionalisation". Rev Geophys, 3, 123 (1965)

14. M Aki: "Maximum Likelihood Estimate of b in the Formula Log(N) = a-bM and its Confidence Limits". Bull Earthquake Res Inst, Tokyo Univ, 43, 237-239 (1965)

15. T Utsu: "A Statistical Significance Test of the Difference in b Value Between Two Earthquake Groups". J Phys Earth, 14, 37-40 (1966)

16. D H Weichert: "Estimation of Earthquake Recurrence Parameters for Unequal Observation Periods for Different Magnitudes". Bull Seism Soc Am, 70, 1337-1346 (1980)

TABLE 1

Summary of results of analysis of the full amplitude ($\text{Log}\frac{A}{T}$) data for selected array stations. The first line for each array gives the parameters determined by the method of Kelly and Lacoss (6) with the 50% threshold μ, standard deviation σ and the seismicity parameters a and b. On the second line are the results using the Page (10) formula to the assumed linear sections (see figures 2 and 3).

	TIME PERIOD Dates (Day,Month,Year) From	To	Years		Number Amplitudes Used	50% Threshold, μ	Standard Deviation, σ	Annual Seismicity, 0.1 m_b interval a	b	Amplitude Range Used For Curve Fits	
NAO	010671	311273	2.58	<	11430	0.04	0.22	2.85	0.83	-0.8	2.3
					4352	-	-	2.90	0.85	0.5	2.3
NB2	010178	311281	4.00	<	8171	0.22	0.24	2.70	0.88	-0.8	3.2
					3934	-	-	2.63	0.84	0.5	3.2
EKA	011081	310683	1.70	<	1537	1.04	0.28	3.52	1.15	0.1	3.0
					164	-	-	3.68	1.22	1.8	3.0
GBA	010479	310683	4.08	<	10320	0.59	0.26	3.17	0.92	-0.6	3.2
					2148	-	-	3.10	0.89	1.2	3.2
WRA	010181	311282	1.77	<	20348	0.26	0.19	3.55	0.91	-0.6	2.8
					6663	-	-	3.54	0.91	0.7	2.8
BMO	010166	311266	1.0	<	3725	0.22	0.18	3.12	1.01	-0.3	2.4
					666	-	-	3.11	1.01	0.9	2.4
CPO	010166	311266	1.0	<	5438	0.52	0.15	(4.31)	(1.64)	-0.3	2.4
					293	-	-	3.42	1.13	1.3	2.4
TFO	010166	311266	1.00	<	12083	-0.04	0.18	(3.62)	(1.41)	-1.0	2.4
					578	-	-	3.12	1.05	0.9	2.4
UBO	010166	311266	1.00	<	11756	0.15	0.17	(3.86)	(1.38)	-0.6	2.4
					602	-	-	3.30	1.02	1.1	2.4
WMO	010166	311266	1.00	<	8734	0.30	0.15	(4.07)	(1.54)	-0.4	2.4
					497	-	-	3.45	1.16	1.1	2.4

TABLE 2

0.1 m_b interval b and a values obtained from ISC amplitude. (Log $\frac{A}{B}$) data computed using the methods of Page (10) and Bender (11). The range of amplitudes over which the fits were made as well as the number of amplitudes within the range are also given.

Station	Page (1968) b	Page (1968) a	Bender (1983) b	Bender (1983) a	Linear Range of $Log_{10}(\frac{A}{T})$		No. $Log\frac{A}{T}$ Values Used
ALE	0.89	2.82	0.90	2.83	2.45	1.35	1330
ALQ	0.99	2.94	0.99	2.92	2.35	1.45	1205
ASP	0.68	2.91	0.69	2.93	2.65	1.65	1595
AVF	1.42	3.49	1.43	3.51	2.75	1.25	671
BKS	1.04	3.43	1.06	3.45	2.95	1.85	852
BMO	0.95	3.02	0.95	3.02	2.95	1.05	2508
BNG	1.08	3.06	1.09	3.07	2.35	1.15	2960
BRG	1.03	3.11	1.04	3.12	2.75	1.35	2059
BSF	1.47	3.56	1.48	3.59	2.65	1.55	391
BUL	1.15	3.16	1.15	3.18	2.55	1.35	1798
CDF	1.09	2.84	1.08	2.84	2.35	1.25	715
CHG	0.75	2.98	0.76	2.99	2.55	1.55	1034
CIR	1.32	3.14	1.33	3.16	2.75	1.35	735
CIK	1.28	3.01	1.30	3.04	2.35	1.25	740
CLL	1.03	3.26	1.03	3.27	2.95	1.65	1767
COL	0.88	3.11	0.87	3.11	2.55	1.35	4542
CPO	0.88	2.92	0.87	2.90	2.75	1.45	1123
DAG	1.06	3.32	1.07	3.33	2.65	1.45	2075
DIX	1.03	2.94	1.04	2.97	2.45	1.45	505
DUG	1.03	2.98	1.03	2.99	2.35	1.35	1207
EDM	1.03	3.46	1.04	3.47	3.05	1.95	1293
EKA	1.05	3.11	1.05	3.12	2.65	1.55	897
EUR	0.96	2.96	0.97	2.96	2.35	1.15	3541
FBA	1.18	3.50	1.18	3.51	2.55	1.55	488
FFC	1.04	3.28	1.04	3.28	2.45	1.35	2417
FLN	1.35	3.55	1.36	3.57	2.65	1.55	557
FUR	1.28	3.88	1.31	3.94	2.65	1.85	896
GBA	1.05	3.28	1.05	3.28	2.45	1.45	595
GIL	1.05	3.44	1.05	3.44	2.65	1.55	2161
GOL	0.89	2.69	0.89	2.68	2.35	1.25	1551
GRF	1.11	3.46	1.11	3.46	2.85	1.85	1080
GRR	1.35	3.59	1.35	3.60	2.95	1.55	636
HAU	1.06	2.87	1.06	2.87	2.45	1.35	652
HFS	1.01	3.39	1.01	3.40	2.55	1.35	4189
HYB	1.00	3.53	1.00	3.54	3.05	2.05	1459
INK	0.94	3.17	0.95	3.20	2.65	1.85	1211
KEV	0.99	3.07	1.00	3.07	3.05	1.65	1344
KHC	0.98	3.07	0.99	3.09	2.75	1.65	1321
KJF	1.20	3.74	1.21	3.76	2.75	1.45	3802
KRA	1.15	3.64	1.16	3.66	2.75	1.75	1766
KRI	1.09	2.81	1.10	2.82	2.45	1.05	573
KRR	1.15	2.86	1.16	2.89	2.35	1.25	656
KTG	0.94	2.91	0.97	2.96	2.45	1.65	820
LAO	0.97	3.25	0.97	3.26	2.55	1.05	3625
LBF	1.51	3.44	1.52	3.46	2.35	1.25	625
LEM	1.02	3.45	1.04	3.49	2.65	1.65	934
LOR	1.06	3.07	1.06	3.07	2.75	1.35	2041
LFF	1.26	3.34	1.26	3.35	2.75	1.35	934
LON	0.95	2.75	0.96	2.77	2.35	1.45	1005
LSF	1.86	4.16	1.87	4.18	2.75	1.45	415
LPO	1.18	3.11	1.19	3.13	2.65	1.45	546
MAT	1.05	3.51	1.06	3.52	3.05	1.65	1141
MBC	0.93	3.21	0.93	3.21	2.35	1.15	4825
MFF	1.37	3.51	1.36	3.50	2.55	1.55	461
MOX	1.05	3.21	1.06	3.22	2.75	1.35	2971
MTD	1.16	2.95	1.18	2.97	2.45	1.45	491
NAO	0.81	2.86	0.82	2.86	2.35	1.05	2285

TABLE 2 (Continued)

Station	Page (1968) b	Page (1968) a	Bender (1983) b	Bender (1983) a	Linear Range of $Log_{10}(\frac{A}{T})$		No. $Log\frac{A}{T}$ Values Used
NB2	1.02	2.99	1.02	2.99	3.05	1.05	1313
NDI	1.18	3.92	1.19	3.96	3.25	2.25	757
NIE	1.00	2.97	1.01	2.99	2.75	1.45	1407
NUR	1.11	3.58	1.12	3.59	2.75	1.55	2744
NVS	0.98	3.36	0.98	3.36	3.35	1.85	296
OBN	1.00	3.52	1.00	3.52	2.95	1.95	293
PMO	1.09	3.58	1.10	3.62	3.15	2.15	619
PMR	1.10	3.51	1.10	3.51	2.85	1.75	1727
PNT	0.94	3.10	0.94	3.10	2.55	1.55	1486
PPN	1.65	4.60	1.65	4.59	2.85	1.95	564
PPT	1.31	4.21	1.31	4.21	3.25	2.25	559
PRE	1.00	2.74	1.00	2.75	2.55	1.45	870
PRU	1.04	3.11	1.05	3.13	3.05	1.85	734
RES	0.90	2.95	0.90	2.95	2.35	1.25	2385
RUV	1.14	3.74	1.14	3.75	3.15	2.05	856
SPA	0.80	3.06	0.81	3.07	2.75	1.65	1589
TIK	0.86	3.13	0.86	3.13	2.75	1.64	408
TLG	0.97	3.22	0.95	3.19	2.45	1.55	405
TPT	1.05	3.49	1.06	3.51	3.25	2.15	631
TSK	0.62	2.46	0.63	2.47	2.35	1.35	875
TUC	0.90	2.75	0.91	2.75	2.45	1.45	938
TUL	0.86	3.06	0.86	3.06	2.75	1.45	3331
UBO	0.99	3.20	0.99	3.21	2.65	1.15	975
UZH	0.89	2.91	0.89	2.93	3.05	1.65	258
VAH	1.04	3.39	1.04	3.38	3.05	1.95	851
WIN	1.09	2.71	1.11	2.74	2.65	1.45	417
WOL	1.15	3.72	1.14	3.69	2.75	1.95	411
WRA	0.85	3.23	0.85	3.23	2.35	1.15	2591
YKC	0.95	2.98	0.94	2.97	2.75	1.55	1026
ZAK	0.80	2.93	0.81	2.94	2.45	1.45	535
ZUL	1.08	3.63	1.09	3.65	2.95	1.85	1137

TABLE 3

ISC seismicity in terms of numbers having magnitudes m_b = 5.8, 5.9 and 6.0 for the period 1964 to 1980 inclusive, for the world and Soviet Union. The values of a (in terms of cumulative numbers) have been computed using these numbers assuming a b value of 0.98. Predicted cumulative counts are based on these a and b values and are for magnitudes as normally rounded to 0.1 m_b unit.

	ISC Seismicity for Number with $5.75 \leq m_b \leq 6.05$		Cumulative Magnitude Frequency Distributions		Predicted Annual Earthquake Counts		
	1964-80	Annually	a_c	b	$m_b \geq 3.5$	$m_b \geq 4.0$	$m_b \geq 4.5$
World	1147±34	67.5±2	7.77	0.98	24500	7900	2600
Soviet Union	103±10	6.1±0.6	6.73	0.98	2200	720	230
Soviet Union Less Kurile-Kamchatka	31±6	1.8±0.3	6.21	0.98	660	210	70

TABLE 4

Individual station estimates of global seismicity as determined by numbers of earthquakes with $m_b \geq 4.0$. Values of C, σ, F_T are used to correct the raw a values to give a_c in terms of global seismicity using equation (11).

Station	Number of Amplitudes	Linear Range for m_b	Station Correction, C	σ	% of Seismicity in Range $\Delta = 30\text{-}90^o$ F_T	b Value	Annual Cumulative a values Raw a	Annual Cumulative a values Corrected a_c	$N\omega$ Number with $m_b \geq 4.0$ using a	$N\omega$ Number with $m_b \geq 4.0$ using a_c
BMO	1321	4.95-6.95	-0.29	0.35	0.45	1.02	7.43	7.93	2517	7888
BNG	1715	5.05-6.25	-0.07	0.50	0.21	1.12	7.83	8.23	2546	6331
BRG	1248	5.25-6.65	-0.11	0.26	0.38	1.08	7.81	8.26	3499	9825
COL	2623	5.25-6.45	0.01	0.33	0.66	0.99	7.56	7.61	4461	4980
DAG	1296	5.35-6.55	-0.02	0.27	0.43	1.23	8.76	9.02	7970	14648
EUR	1841	5.05-6.25	-0.24	0.40	0.46	1.16	8.07	8.44	3076	7178
FFC	1708	5.15-6.65	0.08	0.29	0.40	1.07	7.82	8.02	3921	6238
GIL	1206	5.45-6.55	-0.04	0.35	0.66	1.19	8.68	8.71	9538	10187
HFS	2998	5.15-6.45	0.05	0.45	0.40	0.98	7.57	7.70	5000	6672
KJF	2353	5.35-6.65	0.09	0.28	0.44	1.18	8.67	8.79	10209	13608
LAO	2063	4.95-6.25	-0.10	0.47	0.40	0.96	7.39	7.65	3962	7208
LOR	1234	5.25-6.65	0.06	0.42	0.37	1.06	7.61	7.75	2648	3658
MBC	3803	4.95-6.65	0.14	0.34	0.44	0.97	7.47	7.57	4350	5422
MOX	1803	5.25-6.65	0.02	0.27	0.38	1.09	7.89	8.19	3841	7643
NAO	1458	4.95-6.25	-0.09	0.29	0.40	0.84	6.69	7.10	2355	5988
NUR	1376	5.55-6.85	0.19	0.30	0.42	1.05	8.00	8.06	7120	8234
RES	1620	5.15-6.55	0.13	0.37	0.44	0.95	7.20	7.29	2802	3455
TUL	1137	5.55-6.85	0.21	0.32	0.31	0.98	7.49	7.68	4159	6437

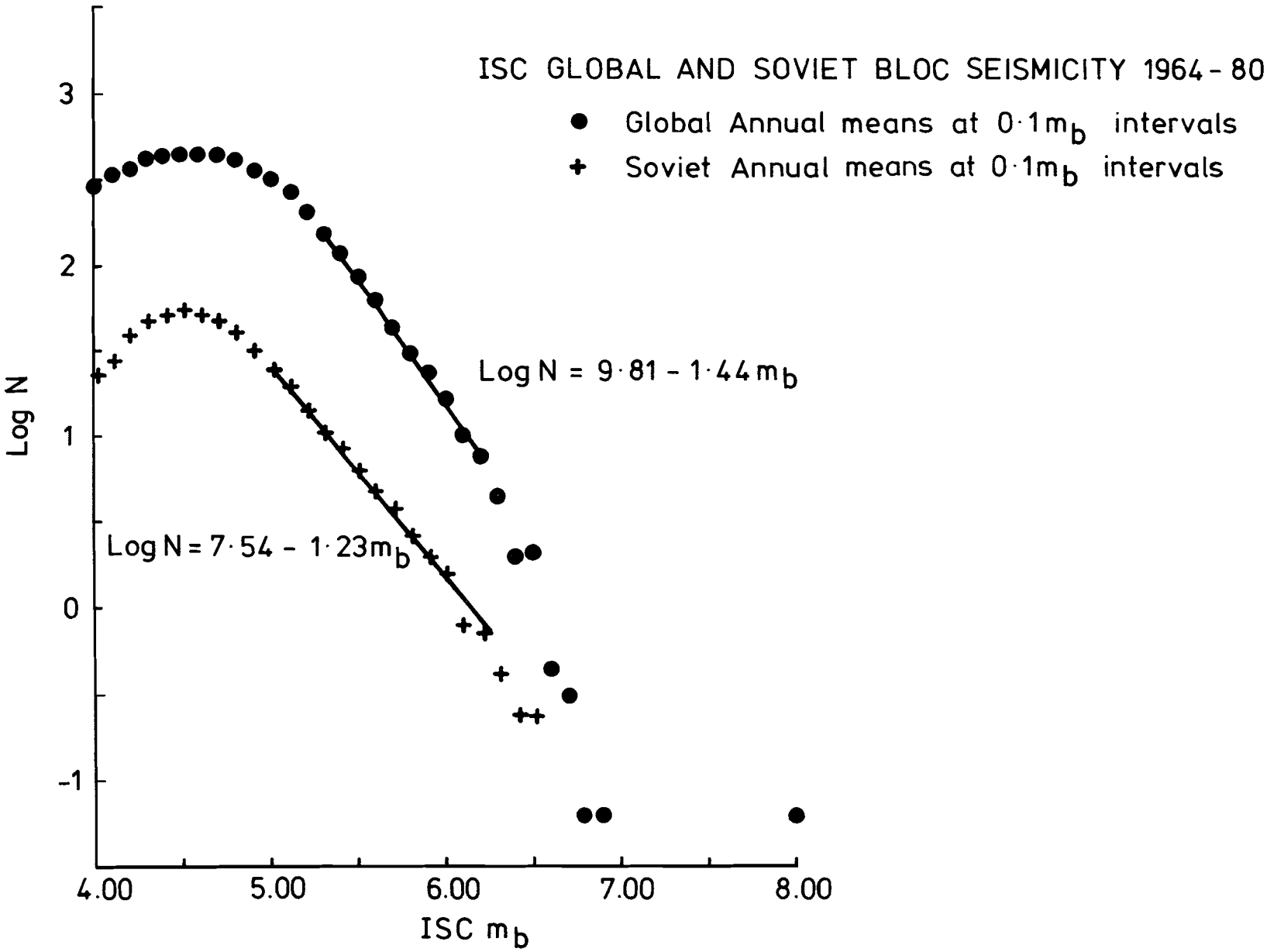

FIGURE 1

ISC magnitudes 1964 to 1980 for whole world and Soviet Bloc plotted as mean annual 0.1 m_b interval counts. Straight lines derived from a and b values are found by method of Bender (11).

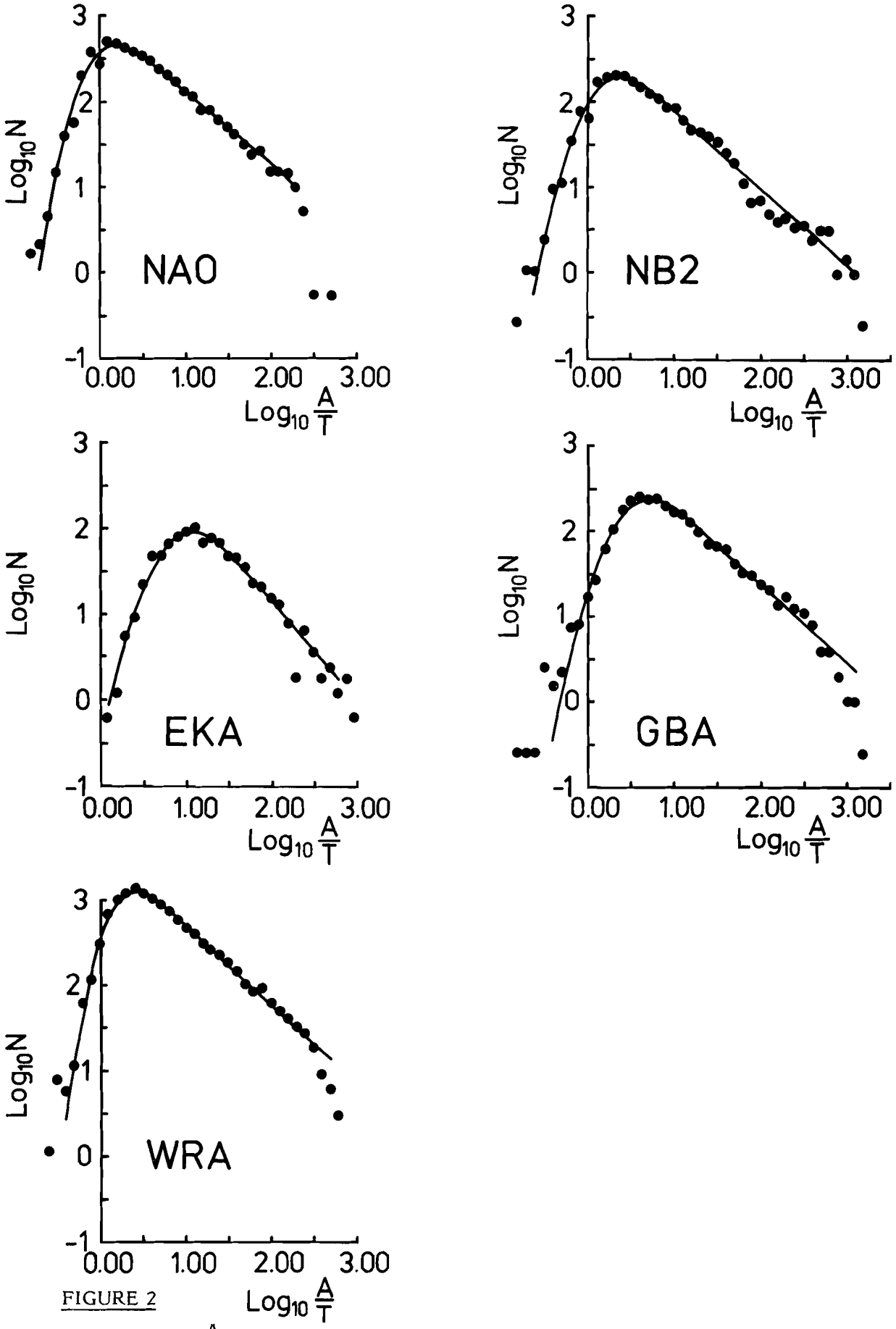

FIGURE 2

Amplitude ($Log\frac{A}{T}$) frequency plots for two operational periods of NORSAR (NAO,NB2) and the United Kingdom arrays EKA, GBA and WRA. Curves are fitted using the method of Kelly and Lacoss (6) and parameters defining the curves are given in table 1.

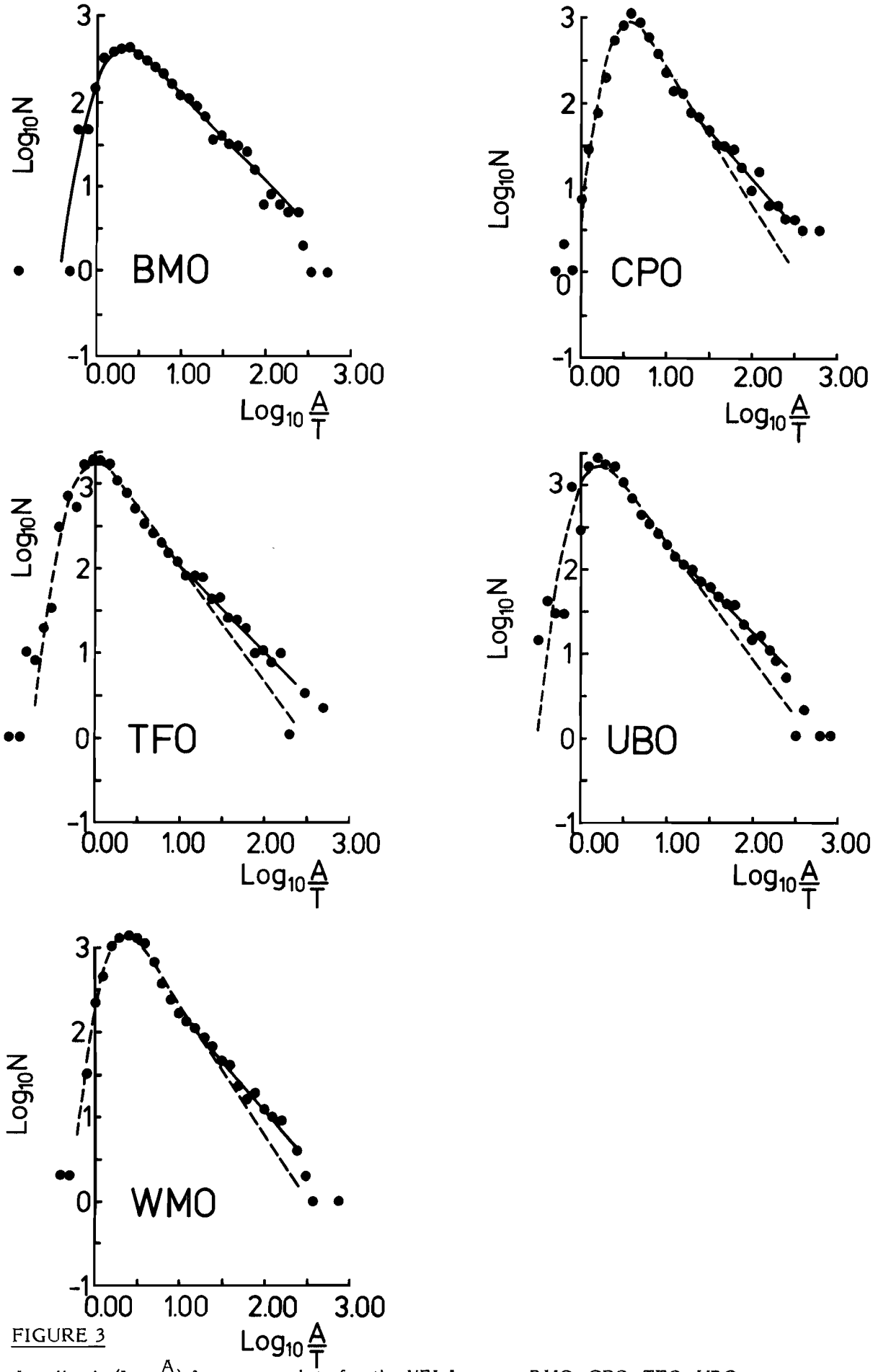

FIGURE 3

Amplitude ($Log \frac{A}{T}$) frequency plots for the VELA arrays BMO, CPO, TFO, UBO, WMO for the year 1966. Dotted curve is fitted using the method of Kelly and Lacoss (6) while the line is a straight linear fit to the assumed linear section using the Page (10) formula. Associated parameters defining the curves and lines are given in table 1.

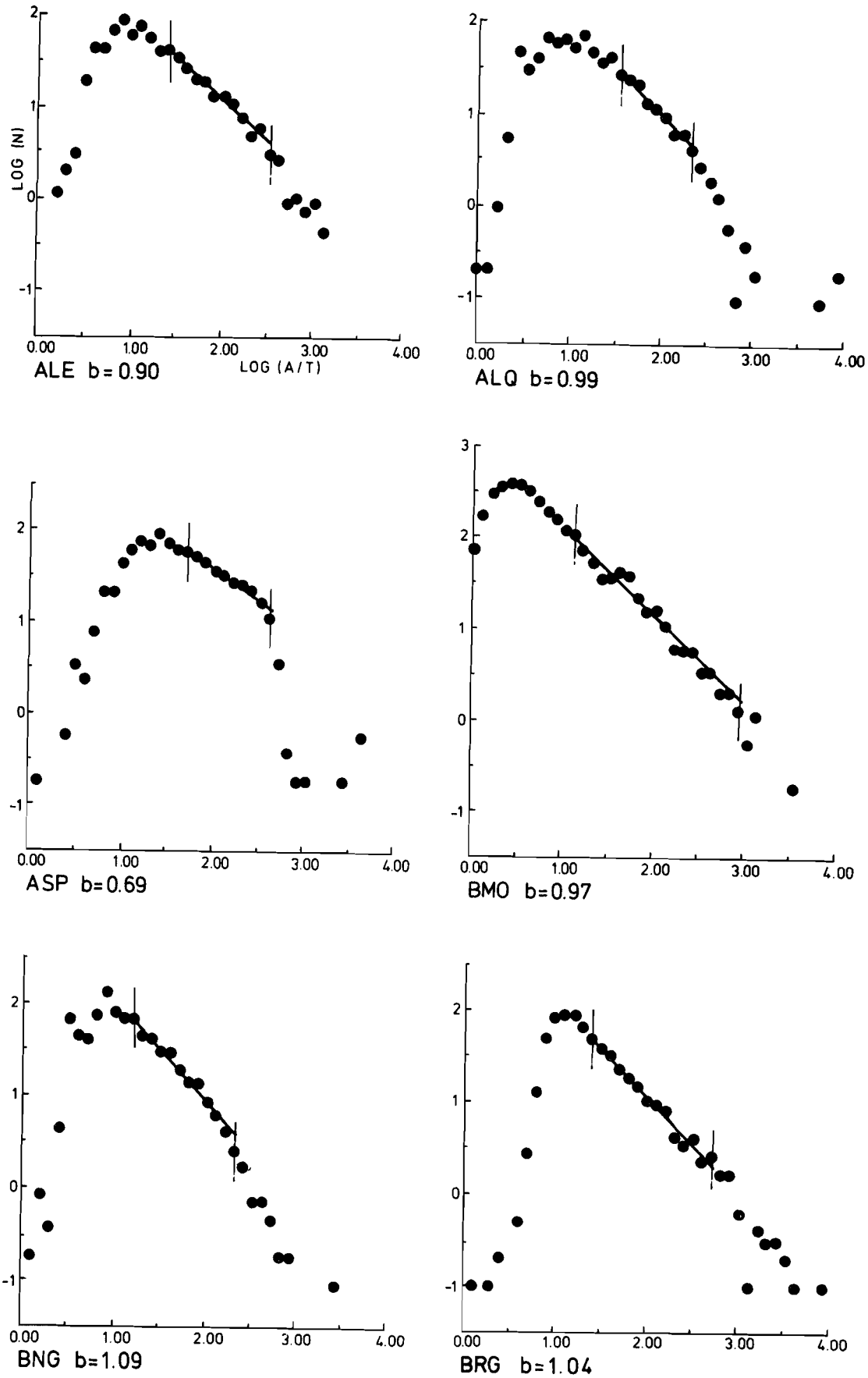

FIGURE 4(a)

Amplitude ($\text{Log}\frac{A}{T}$) frequency plots for 44 stations contributing over 1000 amplitude readings. Fits to the linear sections (as defined by the length of the lines) are by the method of Bender (11). Full details of the results for the full set of stations considered are given in table 2.

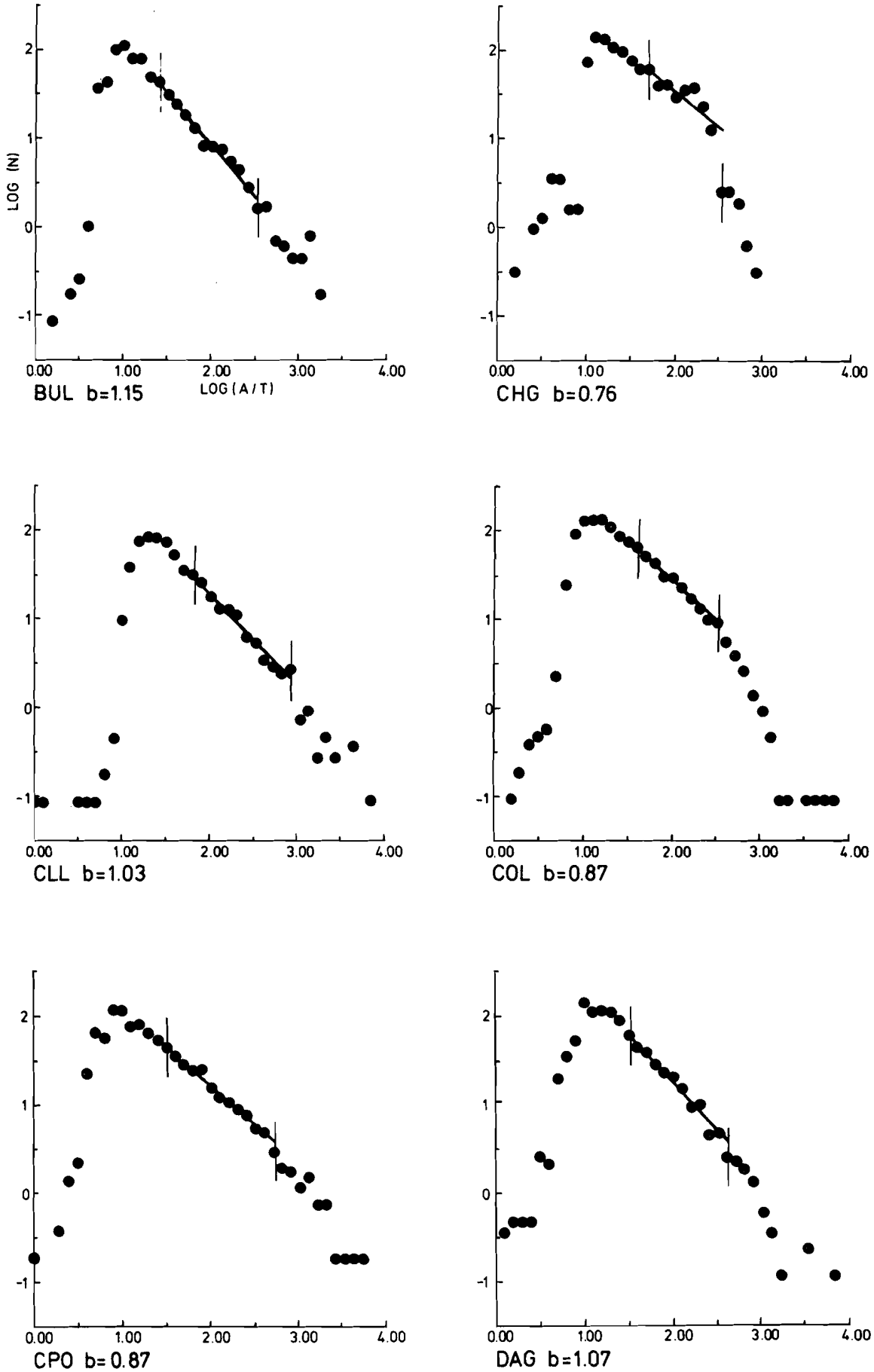
LOG (N)
LOG (A/T)
BUL b=1.15
CHG b=0.76
CLL b=1.03
COL b=0.87
CPO b=0.87
DAG b=1.07

FIGURE 4(b)

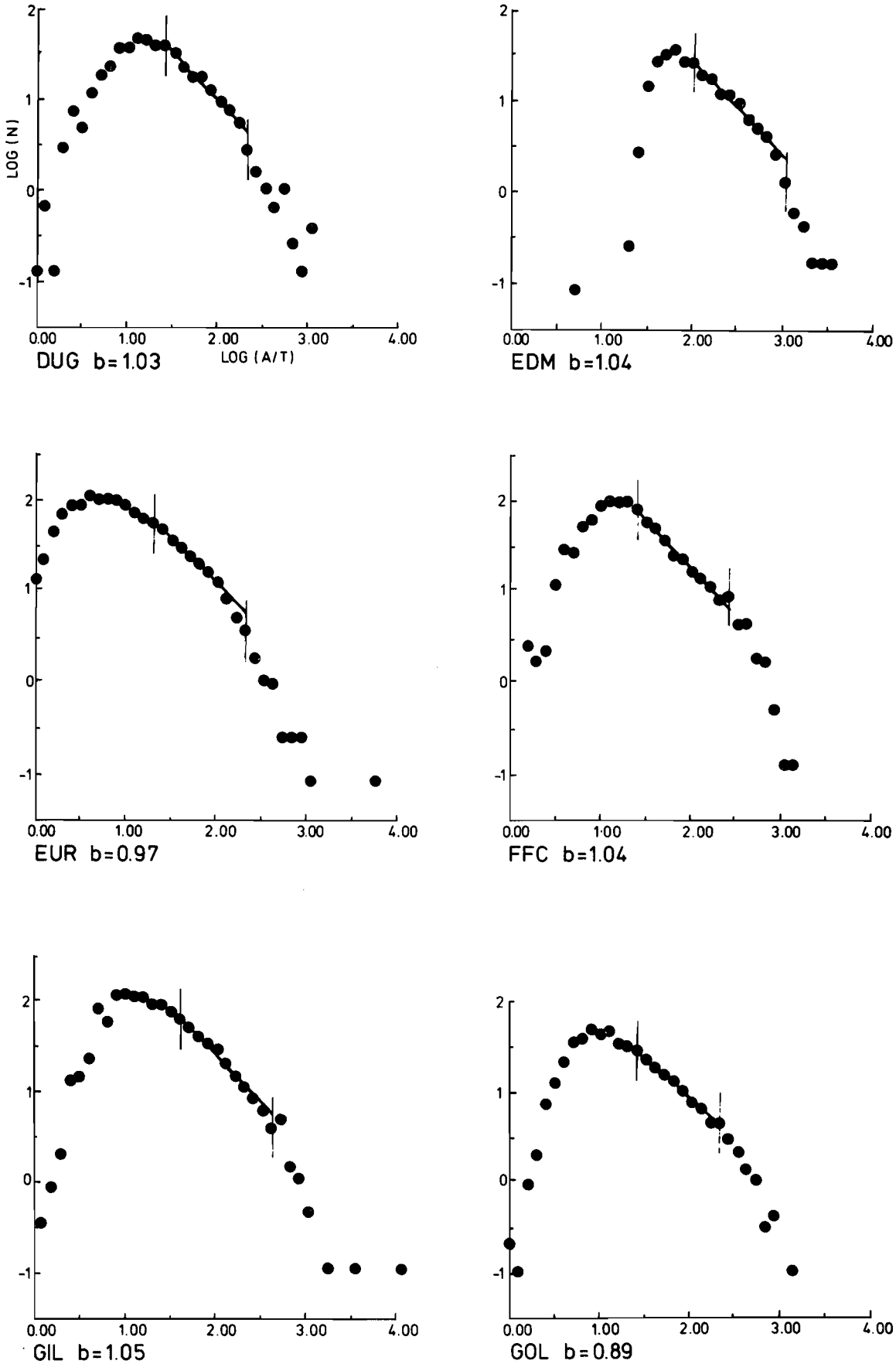
LOG (N)
LOG (A/T)
DUG b=1.03
EDM b=1.04
EUR b=0.97
FFC b=1.04
GIL b=1.05
GOL b=0.89

FIGURE 4(c)

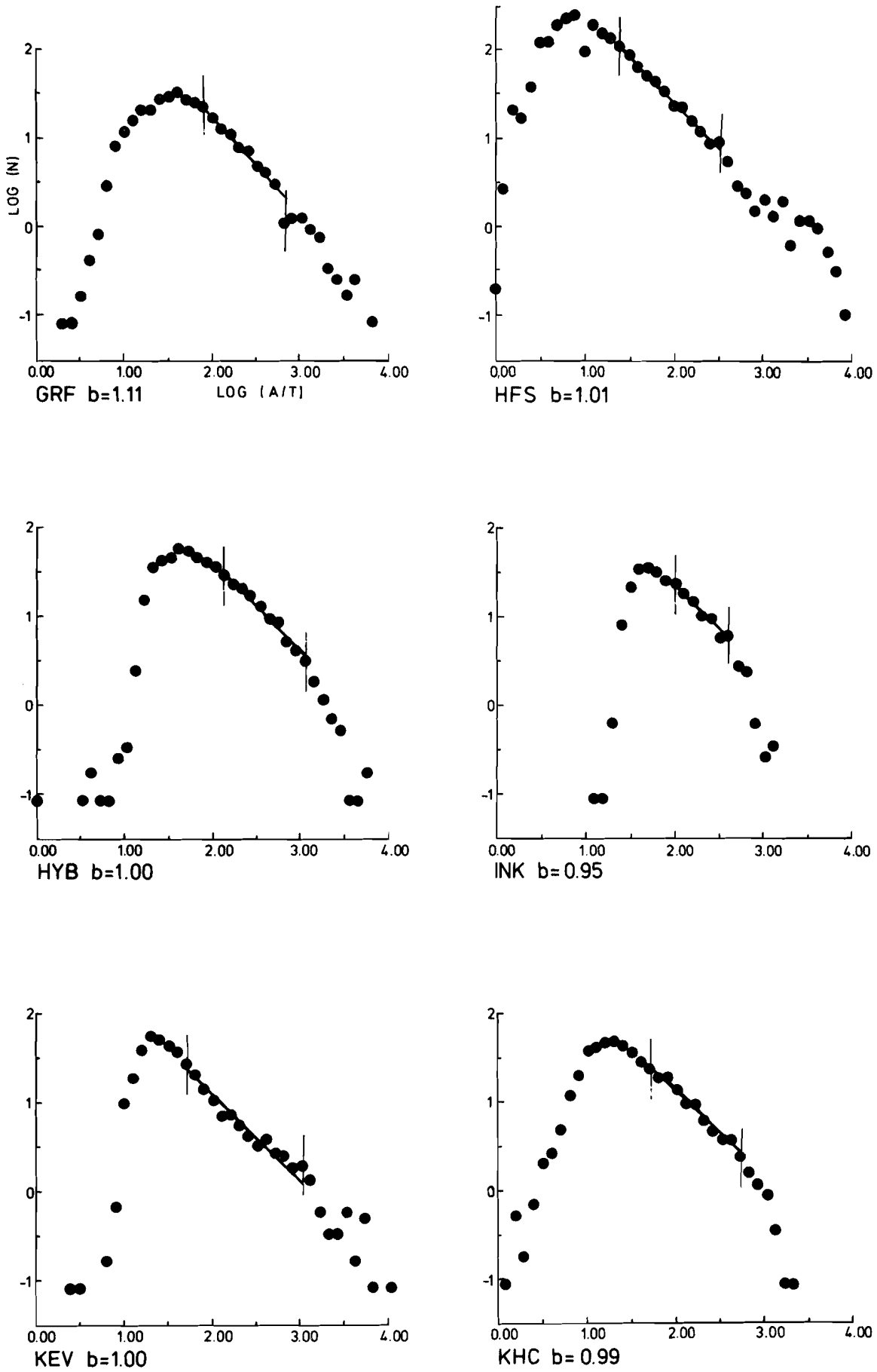
LOG (N)
LOG (A/T)
GRF b=1.11
HFS b=1.01
HYB b=1.00
INK b=0.95
KEV b=1.00
KHC b=0.99

FIGURE 4(d)

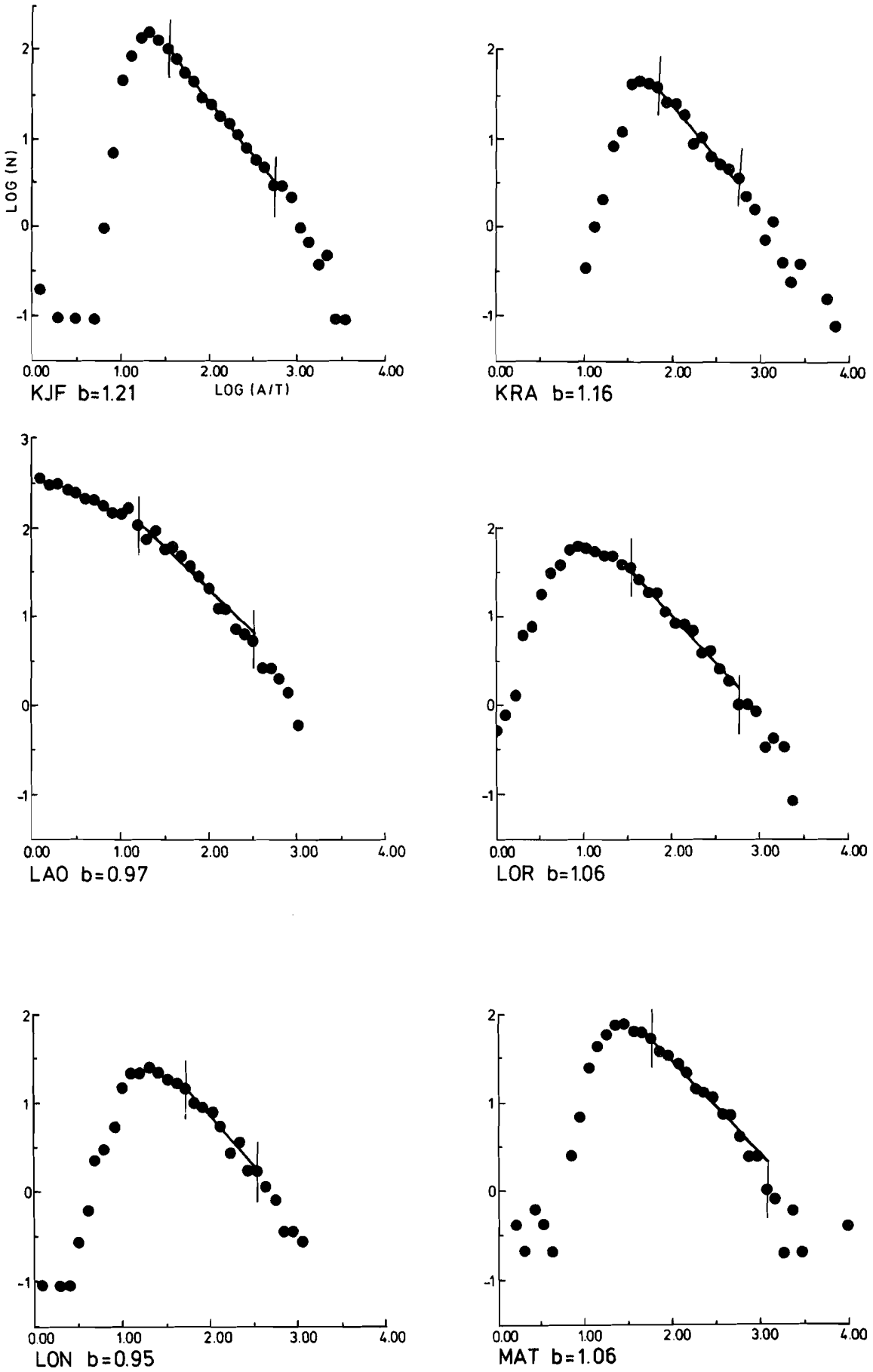
LOG (N)
LOG (A/T)
KJF b=1.21
KRA b=1.16
LAO b=0.97
LOR b=1.06
LON b=0.95
MAT b=1.06

FIGURE 4(e)

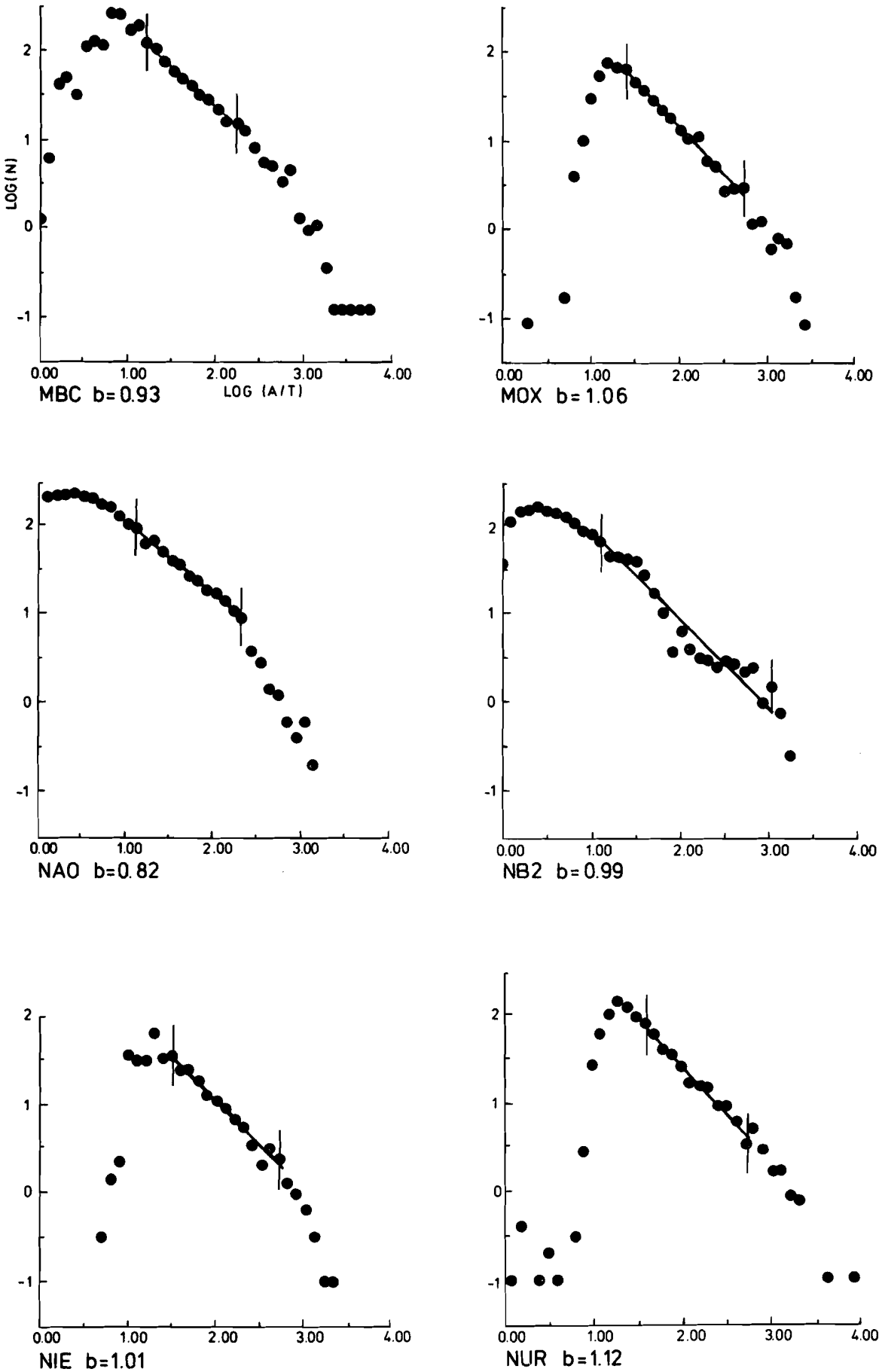
LOG(N)
LOG (A/T)
MBC b=0.93
MOX b=1.06
NAO b=0.82
NB2 b=0.99
NIE b=1.01
NUR b=1.12

FIGURE 4(f)

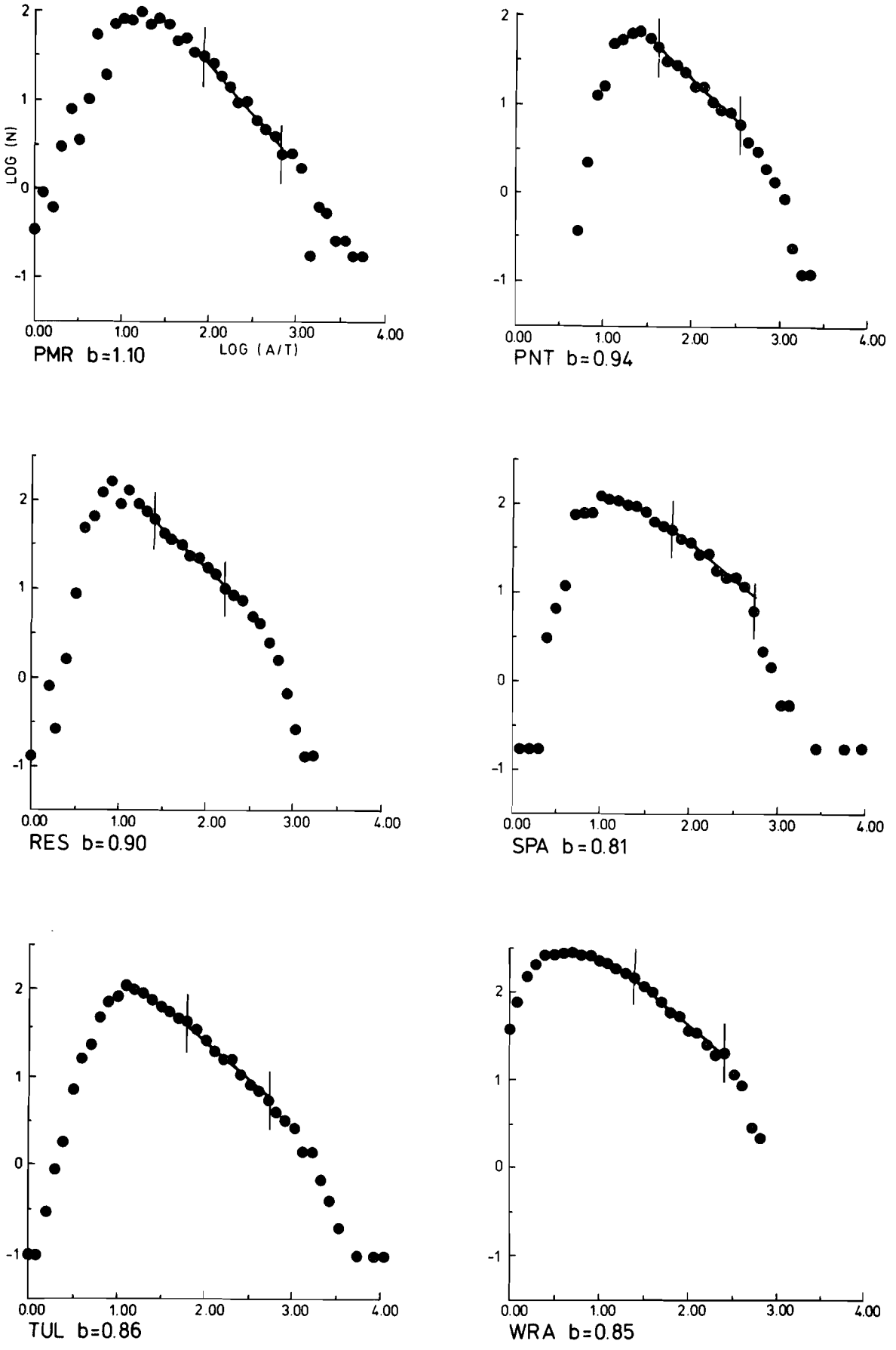
LOG (N)
LOG (A/T)
PMR b=1.10
PNT b=0.94
RES b=0.90
SPA b=0.81
TUL b=0.86
WRA b=0.85

FIGURE 4(g)

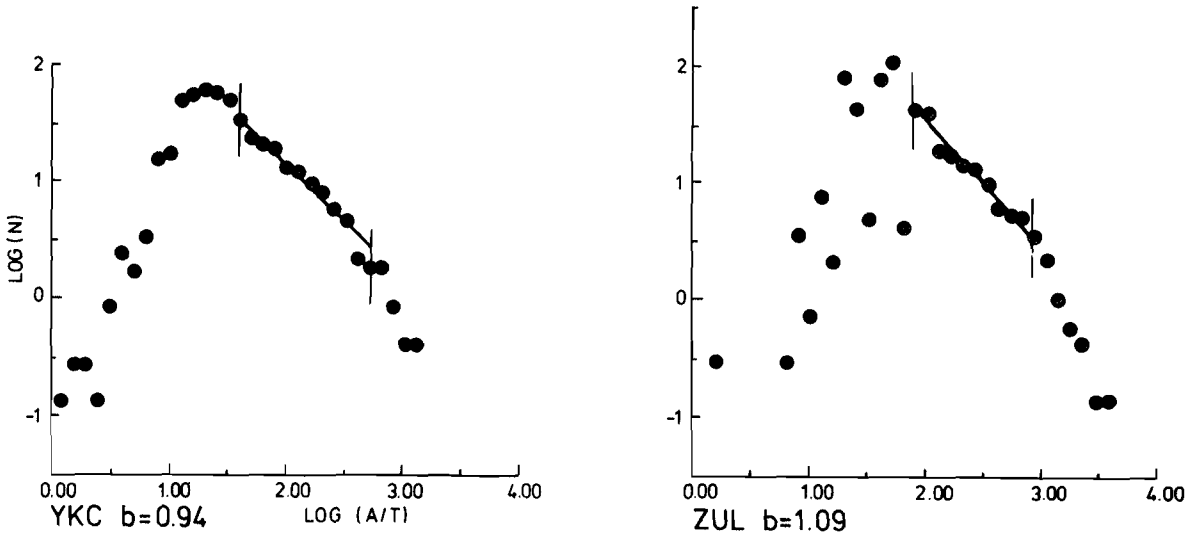
2
1
0
-1
LOG (N)
0.00
1.00
2.00
3.00
4.00
YKC b=0.94
LOG (A/T)
2
1
0
-1
0.00
1.00
2.00
3.00
4.00
ZUL b=1.09

FIGURE 4(h)

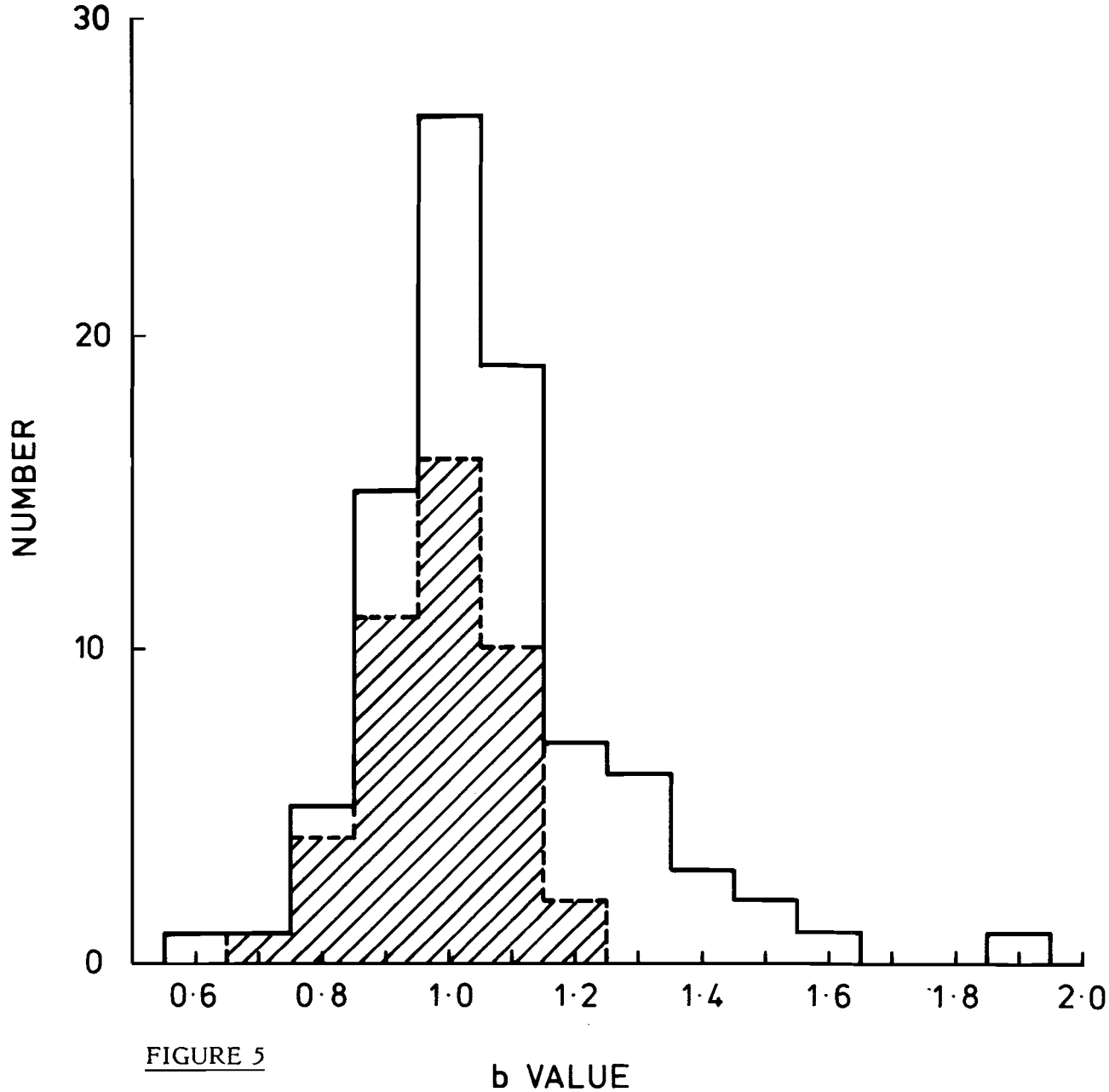

FIGURE 5

Histogram of the distribution of b values based on ISC data for 88 stations. Shaded section is for 44 b values based on 1000 amplitudes or more.

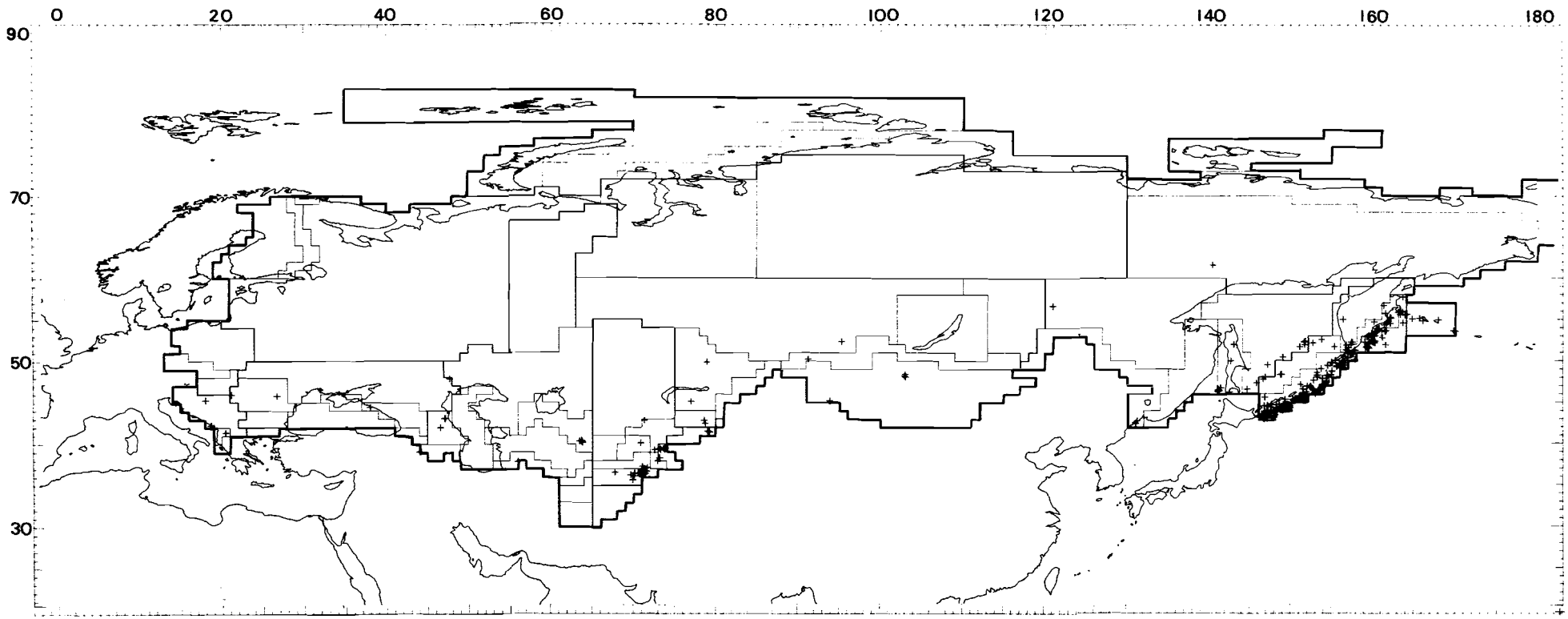

FIGURE 6

Soviet Bloc as defined by the region numbers given in appendix A.

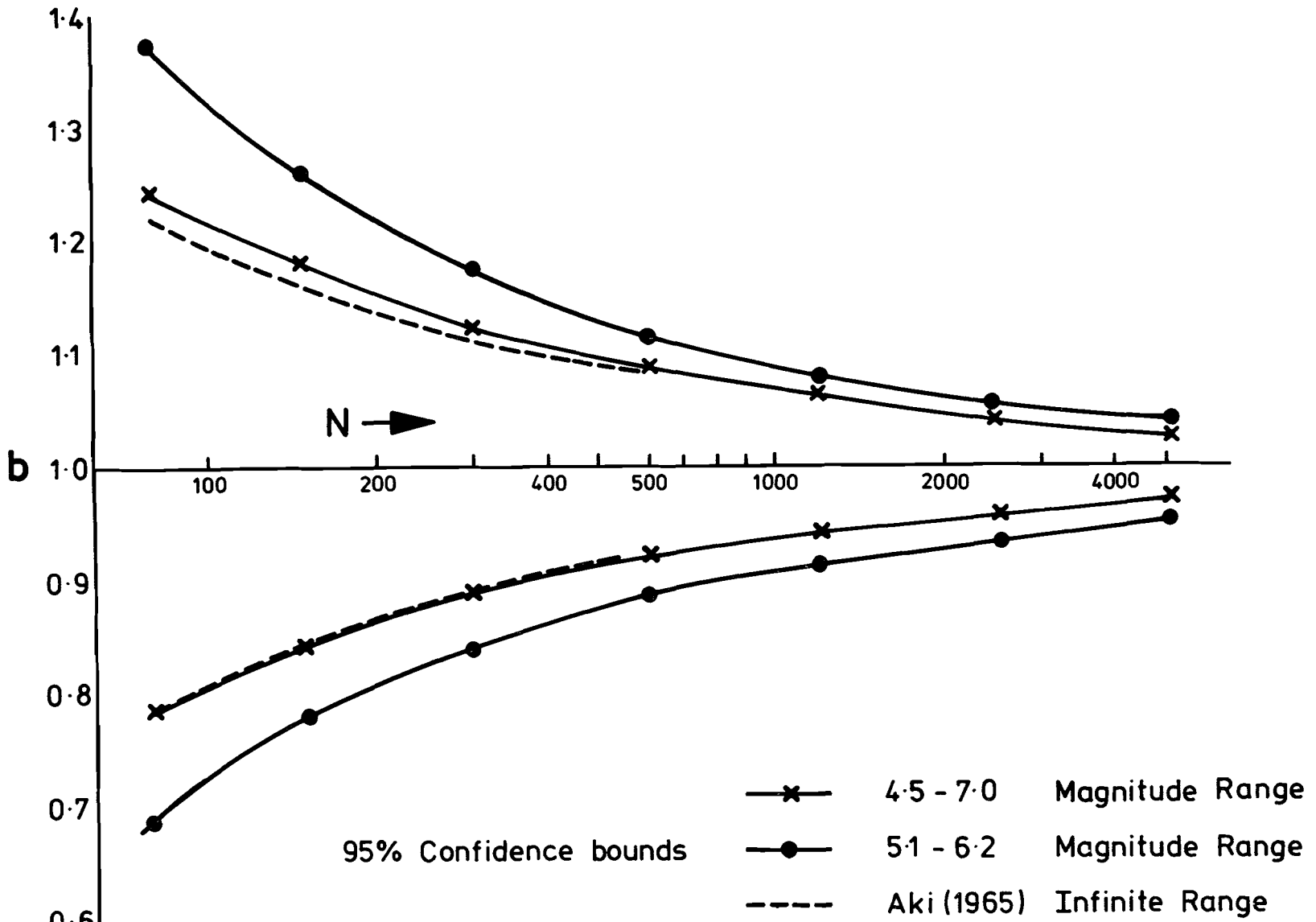

FIGURE 7

95% confidence limits as a function of sample size on b values inferred from simulation studies on data with a population b value of 1.0. Results are shown for data from within a short (1.1 m_b) and larger (2.5 m_b) magnitude range and also the result predicted by Aki (14) which assumes no upper bound on magnitude.

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7. Title

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Seismic detection

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Abstract

A knowledge of the number of earthquakes occurring both regionally and globally is an important consideration in the assessment of the resources required to monitor a Comprehensive Test Ban Treaty using seismological methods. Of particular importance is the number of earthquakes with magnitude (m_b) of 4.0 and greater. This magnitude is near the detection threshold of the most sensitive seismic stations operated to date and corresponds to explosions in granite with yields of 1 to 4 ktons.