United Kingdom Atomic Energy Authority

ATOMIC WEAPONS RESEARCH ESTABLISHMENT

AWRE REPORT NO. 0-54/66

A Special Purpose Least Squares Program

A. Douglas

Recommended for issue by

H. I. S. Thirlaway, Group Leader

Approved by

K. D. Bomford, Deputy Chief of Applied Physics

C13

550.34:519.281
519.281:550.34
681.3:550.34
623.454.8

TABLE OF CONTENTS

SUMMARY

The relative size, M(I,J), of the seismic signal recorded at station I from the Jth explosion at a particular firing site is assumed to be given by the equation

$$M(I,J) = B(J) + S(I) + \varepsilon(I,J),$$

where B(J) depends on the size of the explosion and S(I) is a station term dependent mainly on the distance of the Ith station from the firing site. M(I,J) is measured from seismic records and will usually be in error; ε (I,J) is the error term.

A least squares program is described for estimating: (1) B(J) and S(I), (2) the confidence limits on these quantities, and (3) the differences, and confidence limits on the differences, between all possible pairs of B(J).

Although written for a specific purpose the method is general and can be used to estimate any quantities that can be expressed as equations of the above type.

1. INTRODUCTION

A seismic event radiates elastic waves through the body of the earth. The relative amplitude of these waves as measured at distant recording stations will be determined by two main effects: (1) the size of the event, and (2) the distance of the recording station from the event. The recording instruments and the geology of the recording station and firing site will also have an effect but for explosions from the same firing site these effects will be constant.

If M(I,J) is a measure of the size of the signal (defined as proportional to the log of the measured amplitude) for the Jth explosion at the Ith station, M(I,J) is given by the equation

$$M(I,J) = B(J) + S(I) + \varepsilon\,(I,J), \qquad \text{........ (1)}$$

where B(J) depends on the seismic size of the explosion and S(I) is a station term dependent mainly on the distance of the Ith station from the firing site, but including any effects due to recording instruments and geology of the recording station. M(I,J) is measured from seismic records and will usually be in error; ε (I,J) is the error term.

The problem is to estimate (1) B(J) and S(I) (none of which are known), (2) the confidence limits of these quantities, and (3) the differences, and confidence limits on the differences, between the explosion terms. This report describes a computer program for solving this problem by least squares. The program was written by Mr. J. B. Young and is currently in use at Blacknest; it has been given the name LSMF - Least Squares Matrix Factorisation - for historical reasons. The program has developed from others designed to solve the same problem; all these have been titled LSMF. This name has therefore been retained even though "least squares matrix factorisation" is not a very informative title.

2. THE MODEL

Consider t explosions (fired at one test site) and r recording stations. For every station that records one of these explosions there will be an equation of type (1). If all stations record all explosions this results in rt equations. The system is apparently over-determined as there are only r + t unknowns; this however is not so. Each equation only defines S(I) + B(J); there are no equations relating two or more station terms or two or more explosion terms. There is then no unique solution; whatever value is given to one station term, S(K) say, can be allowed for by adjustments to each of the remaining S(I) and B(J) - equation (1) can always be satisfied.

Further assumptions must then be made. The simplest of these is to give one station term a fixed value. As only the relative size of S(I) and B(J) are really important, M(I,J) being a relative value, this would be acceptable except that confidence limits cannot be determined for the S(I) that is assigned the particular value.

To overcome this difficulty equation (1) is rewritten as

$$M(I,J) = B(J) + S(I) + MBAR + \varepsilon(I,J), \qquad \ldots\ldots\ (2)$$

where MBAR is a constant. As M(I,J) is a purely relative value, the addition of this constant does not materially affect the model. The further assumption is now made that $\sum_J B(J) = 0$ and $\sum_I S(I) = 0$. MBAR can be thought of as the size of the average explosion at the average station; B(J) and S(I) then become corrections to this average for the particular explosion J and station I.

If it is assumed that the errors $\varepsilon(I,J)$ are normally distributed with zero mean and variance σ^2, this model is the same as the widely used analysis of variance model.

3. THE ANALYSIS OF VARIANCE APPROACH

The model described above is simply that of a two way analysis of variance. The data displayed in the usual analysis of variance table are:-

	B(1)	B(2)		B(t)
S(1)	M(1,1)	M(1,2)	. . . . . .	M(1,t)
S(2)	M(2,1)	M(2,2)	. . . . . .	M(2,t)
S(3)	.	.		.
.	.	.	.	.
S(r)	M(r,1)	.	. . . . . .	M(r,t)

Now as $\sum_I S(I)=0$ and $\sum_J B(J) = 0$ and the expectation of $\varepsilon(I,J) = 0$, the average of each column is an estimate of B(J) and the average of each row is an estimate of S(I). The mean value over all M(I,J) gives the value of MBAR. Substituting for S(I), B(J) and MBAR in equation (2) gives the errors $\varepsilon(I,J)$; from these errors σ^2 can be estimated and hence the confidence limits obtained.

Unfortunately this method cannot be applied directly because not all M(I,J) are known - stations of low sensitivity fail to record the smaller events and some records are simply not available. The method of least squares however does not require that all M(I,J) be known.

4. THE METHOD OF LEAST SQUARES

Consider the equation

$$y = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \dots\dots\dots\dots a_nx_n, \qquad \dots(3)$$

where $x_1 \dots\dots x_n$ are independant variables; $a_1, a_2, a_3 \dots\dots a_n$ are unknown coefficients, called the regression coefficients, and y is the dependent variable determined experimentally. Ideally $a_1, a_2 \dots\dots a_n$ can be found simply by observing n values of y for different values of the independent variables and solving the resulting equations.

Usually, however, the measured value of y will be in error and the problem becomes one of estimating the most probable values of a_1, $a_2 \dots\dots a_n$ given $m > n$ values of y. This can be done using the principle of least squares which states: if $\varepsilon_1, \varepsilon_2 \dots\dots \varepsilon_n$ are the errors in m different equations of type (3) the most probable values of a_1, a_2, a_3 ......a_n can be found by making $\varepsilon_1^2 + \varepsilon_2^2 + \ldots\ldots \varepsilon_m^2$ (i.e., the sum of the squared errors) a minimum,

$$\text{or} \quad \frac{\partial \sum_i^m \varepsilon_i^2}{\partial a_j} = 0 \text{ for } j = 1, n.$$

If n = 2 and x_1 is held constant at 1 the problem reduces to the familiar fitting of the "best" straight line.

Suppose that the m equations of type (3) are as follows:-

$$y_1 = a_1 x_{11} + a_2 x_{12} + a_3 x_{13} + \ldots\ldots a_n x_{1n} + \varepsilon_1$$

$$y_2 = a_1 x_{21} + a_2 x_{22} + a_3 x_{23} + \ldots\ldots a_n x_{2n} + \varepsilon_2$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$y_m = a_1 x_{m1} + a_2 x_{m2} + a_3 x_{m3} \quad \ldots\ldots a_n x_{mn} + \varepsilon_m.$$

These equations are called the equations of condition.

$$\text{Now } \sum_i \varepsilon_i^2 = (a_1 x_{11} + a_2 x_{12} + a_3 x_{13} + \ldots\ldots a_n x_{1n} - y_1)^2$$
$$+ (a_1 x_{21} + a_2 x_{22} + \ldots\ldots a_n x_{2n} - y_2)^2$$
$$+ \ldots\ldots$$
$$+ (a_1 x_{m1} + a_2 x_{m2} + a_3 x_{m3} \ldots\ldots a_n x_{mn} - y_m)^2$$

$$\text{and } \frac{\partial \sum_i \varepsilon_i^2}{\partial a_1} = 2\{(x_{11} x_{11} + x_{21} x_{21} + x_{31} x_{31} + \ldots\ldots x_{m1} x_{m1}) a_1$$
$$+ (x_{11} x_{12} + x_{21} x_{22} + \ldots\ldots x_{m1} x_{m2}) a_2$$
$$+ (x_{11} x_{1n} + x_{21} x_{2n} + \ldots\ldots x_{m1} x_{mn}) a_n$$
$$- (x_{11} y_1 + x_{21} y_2 + \ldots\ldots x_{m1} y_m)\}.$$

For the best estimate of a_1, $\frac{\partial \sum_i \varepsilon_i^2}{\partial a_1} = 0$,

$$\text{i.e., } a_1 \sum_i (x_{i1})^2 + a_2 \sum_i x_{i1} x_{i2} + \ldots\ldots a_n \sum_i x_{i1} x_{in} = \sum_i x_{i1} y_i. \quad \ldots.(4)$$

The process of deriving equation (4) is equivalent to multiplying each equation of condition by its own coefficient of a_1; the coefficient of a_j in equation (4) is then the sum of the coefficients of a_j in these new equations. Similar equations are obtained (equivalent to equating $\frac{\partial \sum_i \varepsilon_i^2}{\partial a_j}$ to zero for j = 2,n) by multiplying each equation of condition by its own coefficient of a_j and summing coefficients. This produces n equations, called the normal equations, with n unknowns. In matrix form the normal equations are

$$\begin{bmatrix} \Sigma x_{i_1} x_{i_1} & \Sigma x_{i_1} x_{i_2} & \cdot & \cdot & \Sigma x_{i_1} x_{in} \\ \Sigma x_{i_2} x_{i_1} & \Sigma x_{i_2} x_{i_2} & \cdot & \cdot & \Sigma x_{i_2} x_{in} \\ \Sigma x_{i_3} x_{i_1} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \Sigma x_{in} x_{i_1} & \cdot & \cdot & \cdot & \Sigma x_{in} x_{in} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_n \end{bmatrix} = \begin{bmatrix} \Sigma x_{i_1} y_i \\ \Sigma x_{i_2} y_i \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Sigma x_{in} y_i \end{bmatrix} \quad \ldots.(5)$$

or XA = Y; the X matrix being symmetrical about the diagonal.

The normal equations can usually be solved uniquely. Any of the usual methods can be used, but one method, matrix inversion, has advantages if the confidence limits of the unknowns are required. Matrix inversion is therefore used in the LSMF program.

If the inverse of matrix X is the matrix C

$$\begin{bmatrix} c_{11} & c_{12} & \cdot & \cdot & \cdot & c_{1n} \\ c_{21} & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{31} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{n1} & \cdot & \cdot & \cdot & \cdot & c_{nn} \end{bmatrix}$$

X and C are related by the equation

$$\begin{bmatrix} c_{11} & \cdot & c_{1j} & \cdot & \cdot & c_{1n} \\ c_{21} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{n1} & \cdot & \cdot & \cdot & \cdot & c_{nn} \end{bmatrix} \begin{bmatrix} \Sigma(x_{i1})^2 & \cdot & \cdot & \Sigma x_{i1}x_{in} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \Sigma x_{in}x_{i1} & \cdot & \cdot & \Sigma x_{in}x_{in} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \cdot & \cdot & 0 \\ 0 & 1 & 0 & \cdot & \cdot & 0 \\ 0 & 0 & 1 & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & \cdot & & & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & 1 \end{bmatrix} \quad \ldots.(6)$$

The elements of the inverse matrix can be found by expanding to give a series of linear equations.

Thus, the result of multiplying the X matrix by the vth row of the inverse matrix (see also Appendix A) is

$$\begin{array}{l} c_{v1}\Sigma x_{i1}x_{i1} + c_{v2}\Sigma x_{i1}x_{i2} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad c_{vn}\Sigma x_{i1}x_{in} = 0 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ c_{v1}\Sigma x_{iv}x_{i1} + c_{v2}\Sigma x_{iv}x_{i2} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad c_{vn}\Sigma x_{iv}x_{in} = 1 \quad \ldots.(7) \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{array}$$

Solving this set of equations for c_{v1}, c_{v2}, ...c_{vn} gives the elements of the vth row of the inverse matrix.

Multiplying equation (5) by C gives

$$A = CY$$

or

$$\begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ a_n \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdot & \cdot & \cdot & c_{1n} \\ c_{21} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{n1} & \cdot & \cdot & \cdot & \cdot & c_{nn} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \quad \ldots.(8)$$

each of the elements of the matrix can then be evaluated as both C and Y are known. The part played by the inverse matrix in determining the confidence limits will be discussed in the next section.

Equation (2) can be put in a form similar to equation (3) as follows:-

$$M(I,J) = 1\ MBAR + 0\ S(1) + \ldots\ldots 1\ S(I) + \ldots\ldots 0\ S(r) +$$

$$0\ B(1) + \ldots\ldots 1\ B(J) + \ldots\ldots 0\ B(t) + \varepsilon(I,J),$$

M(I,J) is now equivalent to y, the dependent variable, MBAR to a_1, S(I) and B(J) to the remaining a's up to a_n and the independent variables are all either 1 to 0.

To include the assumptions $\sum_{I}^{r} S(I) = 0$ and $\sum_{J}^{t} B(J) = 0$ two further equations of conditions have to be added:

$$0 = 0\ MBAR + 1\ S(1) + 1\ S(2) + \ldots\ldots 1\ S(I) + \ldots\ldots 1\ S(r)$$

$$+ 0\ B(1) + \ldots\ldots 0\ B(J) \ldots\ldots + 0\ B(t)$$

and

$$0 = 0\ MBAR + 0\ S(1) + 0\ S(2) + \ldots\ldots 0\ S(J) + \ldots\ldots 0\ S(r)$$

$$+ 1\ B(1) + \ldots\ldots 1\ B(J) \ldots\ldots 1\ B(t).$$

Using these equations of condition the normal equations can be derived in exactly the same way as described above.

As an example consider the following set of equations of condition:

$$\begin{bmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&1&0&0&1 \\ 0&0&1&1&0&0&1 \\ 1&0&0&0&1&0&1 \\ 0&0&1&0&1&0&1 \\ 1&0&0&0&0&1&1 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&0&1&1 \\ 0&0&0&1&1&1&0 \\ 1&1&1&0&0&0&0 \end{bmatrix} \begin{bmatrix} S(1) \\ S(2) \\ S(3) \\ B(1) \\ B(2) \\ B(3) \\ MBAR \end{bmatrix} = \begin{bmatrix} 2.0 \\ 3.0 \\ 4.0 \\ 4.0 \\ 4.0 \\ 4.0 \\ 5.0 \\ 7.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

In this example it is assumed that station S(2) did not record explosion B(2).

Multiplying each equation of condition by its own coefficient of S(1) and summing coefficients gives the first normal equation

$$4\ S(1) + S(2) + 1\ S(3) + 1\ B(1) + 1\ B(2) + 1\ B(3) + 3\ MBAR = 10.0.$$

Multiplying each equation of condition by its own coefficient of S(2) and summing gives

$$1\ S(1) + 3\ S(2) + 1\ S(3) + 1\ B(1) + 0\ B(2) + 1\ B(3) + 2\ MBAR = 8.00.$$

Similar normal equations can be obtained for S(3), B(1), B(2), B(3) and MBAR.

In matrix form the equations are

$$\left[\begin{array}{ccc|ccc|c} 4&1&1&1&1&1&3\\ 1&3&1&1&0&1&2\\ 1&1&4&1&1&1&3\\ \hline 1&1&1&4&1&1&3\\ 1&0&1&1&3&1&2\\ 1&1&1&1&1&4&3\\ \hline 3&2&3&3&2&3&8 \end{array}\right] \begin{bmatrix} S(1)\\ S(2)\\ S(3)\\ B(1)\\ B(2)\\ B(3)\\ MBAR \end{bmatrix} = \begin{bmatrix} 10.0\\ 8.0\\ 15.0\\ 9.0\\ 8.0\\ 16.0\\ 33.0 \end{bmatrix} \qquad \ldots\ldots (9)$$

A series of linear equations has a unique solution if the determinant of the coefficients of the unknowns is not zero. The determinant of the matrix of coefficients in equation (9) is non-zero, but if equation (1) is used as the model it can easily be shown that the determinant of coefficients is zero. Thus, the normal equations now become

$$\begin{bmatrix} 3 & 0 & 0 & 1 & 1 & 1 \\ 0 & 2 & 0 & 1 & 0 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 & 0 & 0 \\ 1 & 0 & 1 & 0 & 2 & 0 \\ 1 & 1 & 1 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} S(1) \\ S(2) \\ S(3) \\ B(1) \\ B(2) \\ B(3) \end{bmatrix} = \begin{bmatrix} 10.0 \\ 8.0 \\ 15.0 \\ 9.0 \\ 8.0 \\ 16.0 \end{bmatrix}$$

Adding row 2 and 3 to row 1 and rows 5 and 6 to row 4 makes the new rows 1 and 4 identical. Subtracting row 4 from row 1 makes all row 1 zero; hence the determinant is zero. This is true of any matrix based on equation (1).

5. CONFIDENCE LIMITS

Estimates of the regression coefficients can be found by solving the normal equations. As a measure of the reliability of these estimates it is useful to compute the limits, called confidence limits, of the range within which the true value of the regression coefficients can be expected to lie with a given probability. The smaller this range turns out to be the more reliable are the estimates.

Consider the simple case of a random variable normally distributed with variance σ^2 and mean ξ, then it is easily shown that any item picked at random from such a population will lie between $\xi + 1.96\sigma$ and $\xi - 1.96\sigma$ (or in words within roughly two standard deviations of the mean) with a 95% probability, i.e., 19 times in 20.

Confidence limits are arrived at in a similar way; the main difference is that ξ and σ are not known and have to be estimated.

The estimates of the regression coefficients are analogous to the mean in the above simple example. To estimate the variance requires a more detailed study of equation (8). Expanding equation (8)

$$a_1 = c_{11}\sum_i^m x_{i1}y_i + c_{12}\sum_i^m x_{i2}y_i \;.\;.\;.\;.\;.\; + c_{1n}\sum_i^m x_{in}y_i$$

$$a_2 = c_{21}\sum_i^m x_{i1}y_i + c_{22}\sum_i^m x_{i2}y_i \;.\;.\;.\;.\;.\; + c_{2n}\sum_i^m x_{in}y_i$$

$$\vdots$$

$$a_v = c_{v1}\sum_i^m x_{i1}y_i + c_{v2}\sum_i^m x_{i2}y_i \;.\;.\;.\;.\;.\; + c_{vn}\sum_i^m x_{in}y_i \qquad \ldots. \; (10)$$

$$\vdots$$

Rearranging (10)

$$a_v = \sum_i^m y_i \left[c_{vi}x_{i1} + c_{v2}x_{i2} + \;.\;.\;.\;.\; c_{vn}x_{in}\right], \qquad \ldots. \; (11)$$

which shows that a_v is a linear function of y_i.

Now the quantity in square brackets in equation (11) is solely a function of the independent variables and can be represented by a single quantity, say k_{vi}.

Then

$$a_v = \sum_i^m y_i k_{vi}$$

and (using equation (B3), Appendix B)

$$V\left[a_v\right] = \sum_i^m k_{vi}^2 V\left[y_i\right] = \sigma^2 \sum_i^m k_{vi}^2,$$

where $V[a_v]$ is understood to mean the variance of a_v and σ^2 is the variance of y_i, i.e., the variance of the errors ε_i. It can also be shown that, because a_v is a linear function of y_i which is normally distributed, a_v will also be normally distributed [1].

Now σ^2 is not known, so $V[a_v]$ cannot be determined. An estimate of σ^2, s^2 say, can however be obtained. Thus,

$$s^2 = \frac{\Sigma(\varepsilon_i')^2}{m - n},$$

where m is the number of equations of condition and n is the number of

---

1. K. A. Brownlee: (1965) "Statistical Theory and Methodology in Science and Engineering". John Wiley and Sons Incorporated, New York.

unknowns (regression coefficients). The quantity m - n is called the number of degrees of freedom. An estimate of the errors ε_i' is obtained by substituting the regression coefficients in the equations of condition.

As a_ν is normally distributed with variance $s^2 \sum_i^m k_{\nu i}^2$, the 95% confidence limits should then be $a_\nu \pm 1.96 \sqrt{s^2 \sum_i^m k_{\nu i}^2}$. This is only true if the degrees of freedom D is very large. For small D, s^2 is a less reliable estimate of σ^2; to allow for this the confidence limits become $a_\nu \pm t \sqrt{s^2 \sum_i^m k_{\nu i}^2}$, where t (called Students t) depends on the degrees of freedom and is > 1.96. (Tables of Students t for various degrees of freedom and level of probability are given in most books on statistics.)

To determine the confidence limits $\sum_i^m k_{\nu i}^2$ must be evaluated. At first sight this appears a formidable task; it can however be shown that $\sum_i^m k_{\nu i}^2$ is simply $c_{\nu\nu}$; the νth diagonal element in the inverted matrix.

This can be demonstrated as follows:-

$$\sum_i^m k_{\nu i}^2 = \sum_i^m k_{\nu i} k_{\nu i}$$

$$= \sum_i^m k_{\nu i}[c_{\nu 1} x_{i1} + c_{\nu 2} x_{i2} + \dots\dots c_{\nu n} x_{in}]$$

$$= c_{\nu 1} \sum_i^m k_{\nu i} x_{i1} + c_{\nu 2} \sum_i^m k_{\nu i} x_{i2} + \dots\dots c_{\nu n} \sum_i^m k_{\nu i} x_{in}.$$

Considering now only the $c_{\nu h}$ term of the right hand side,

$$c_{\nu h} \sum_i^m k_{\nu i} x_{ih} = c_{\nu h} \sum_i^m [c_{\nu 1} x_{i1} + \dots\dots c_{\nu n} x_{in}] x_{ih}$$

$$= c_{\nu h} [c_{\nu 1} \Sigma x_{i1} x_{ih} + c_{\nu 2} \Sigma x_{i2} x_{ih} + \dots\dots$$

$$\dots\dots c_{\nu n} \Sigma x_{in} x_{ih}].$$

When h = ν the quantity in the square brackets is identical to the left hand side of an equation formed by multiplying the νth row of the C matrix by the νth column of the X matrix; from (6) this is equal to 1. For h $\neq \nu$ the quantity in square brackets is identical to the left hand side of one of the other equations (6); the right hand side of all these equations is zero.

Thus, $\sum_i^m k_{\nu i}^2 = c_{\nu\nu}$, the νth diagonal element of the inverted matrix.

and $V[a_\nu] = c_{\nu\nu} s^2$; it is because of this that matrix inversion is used for solving the normal equations.

To get the confidence limits on the difference of two a's, a_v and a'_v the variance on the difference is required, i.e., $V[a_v - a'_v]$. Now $V[a_v - a'_v] = V[a_v] + V[a'_v] - 2\,Cov[a_v, a'_v]$, where $Cov[a_v, a'_v]$ is the covariance of a_v, a'_v (for proof see Appendix B). By an analysis similar to that given for $V[a_v]$ it can be shown that

$$Cov[a_v, a'_v] = s^2 c_{vv'},$$

i.e., the product of the variance of the errors and the element of the inverse matrix that lies at the intersection of the vth row and the v'th column (or vice versa - the two elements $c_{vv'}$, and $c_{v'v}$ are equal because the inverse matrix is also symmetrical).

The variance of the differences of two a's is then given by

$$V[a_v - a'_v] = s^2(c_{vv} + c_{v'v'} - 2c_{vv'}).$$

The 95% confidence limits of $a_v - a'_v$ is then $t\sqrt{V[a_v - a'_v]}$.

Confidence limits on MBAR, S(I), B(J) and on the differences between each pair of explosion terms (B(J's)), are calculated by the methods outlined above. As the M(I,J)'s are only relative values the confidence limits on the absolute values of MBAR, S(I) and B(J) have little meaning. The confidence limits on the differences of the explosion terms are however valuable as they are confidence limits on the absolute differences between the seismic sizes of pairs of explosions.

6. WEIGHTING

So far it has been assumed that the errors in the dependent variables all have the same variance σ^2. This may not be so; some measurements may be known with greater (absolute) accuracy. To get the best estimate of the regression coefficients, i.e. the one with minimum variance, each equation of condition should be weighted by a factor $\sqrt{w}$, where $w = \frac{1}{\sigma_i^2}$; σ_i^2 is the variance of the ith measurement (for a discussion of weighting see reference [1]). A facility for weighting any equation of condition has therefore been included in the LSMF program, although σ_i^2 will usually be difficult to estimate.

7. THE PROGRAM

The program will accept data for up to 60 explosions recorded at a maximum of 200 stations. The input to the program is:-

(1) Students t tables for the 95% probability level.

(2) A title for the data being processed and any number of comment cards.

1. E. Whittaker and G. Robinson: (1944) "The Calculus of Observations: A Treatise on Numerical Mathematics". Fourth Edition, Blackie, London and Glasgow

(3) An identification code for each station.

(4) An identification code for each explosion.

(5) Cards with station code, event code, M(I,J) and the weight to be assigned to M(I,J). If the weight is left blank it is taken to be unity.

The matrix of coefficients of the normal equations (the X matrix) are then set up. This could be done as outlined in Section 4, i.e., by setting up all the equations of condition then multiplying each equation by the coefficient of each unknown in turn and summing. This would need a large amount of storage space in the machine to little purpose as most of the terms in the equations of condition are zero (cf, the example given in Section 4). A method of constructing the matrix of coefficients directly has therefore been devised. As a result of this a far larger number of unknowns (260) can be handled by the program than would be possible if the equations of condition had to be stored in their entirety.

The X matrix is set up as follows: first the whole matrix is zeroed. Each non-zero element of the X matrix is now computed from the input data and stored in its appropriate place in the matrix. As the resultant matrix is symmetrical about the diagonal only the upper triangular matrix and the diagonal elements have to be computed. The upper triangular elements, X_{ij}, are then repeated in the lower triangular position X_{ji}. For the purposes of construction the matrix can be divided into 7 parts (see Figure 1). The equivalent 7 parts are shown in dotted outline in equation (9)).

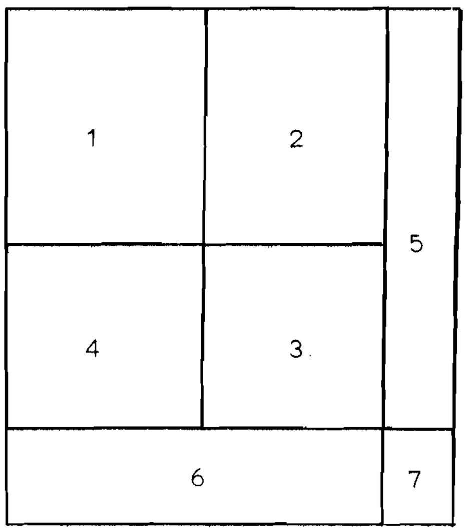

FIGURE 1

If t is the number of explosions read in and r the number of stations, part 1 is an r × r matrix, part 3 a t × t matrix, parts 2 and 4 are r × t matrices, part 2 having t columns and r rows and part 4, r columns and t rows, parts 5 and 6 are (r + t) × 1 matrices and part 7 is a single element.

Matrix 2 is constructed first. The first element in the first row of this matrix is unity if S(1) is recorded at station B(1), the second element is unity if S(1) recorded B(2) and so on to B(t). Similarly in the second row the first element is unity if S(2) recorded B(1), the second element if S(2) recorded B(2) and so on. Rows 3, 4, ... t of matrix 2 are constructed in a similar way. Matrix 4 is now constructed by reflecting matrix 2 in the diagonal.

The diagonal elements of matrix 1 are now formed by summing the corresponding row of matrix 2 and the diagonal elements of matrix 3 by summing the corresponding rows of matrix 4. The diagonal elements of 1 and 3 are now repeated in order in the column matrix 5 and the row matrix 6. Element 7 is half the sum of the elements in the column matrix 5. Finally 1 is added to each element of matrices 1 and 3.

The above somewhat involved process produces the correct matrix of normal equations with $(r + t + 1)^2$ elements.

The first element of the right hand side of equation (5) is formed by summing all M(1,J) (it is assumed that M(I,J) = 0 if station I did not record explosion J), the second element by summing all M(2,J) and so on. Element r of the right hand side is then $\sum_J M(r,J)$. Element r + 1 is $\sum_I M(I,1)$ element r + 2 is $\sum_I M(I,2)$ and so on. Element r + t + 1 is $\sum_{IJ}\sum M(I,J)$.

The setting up of the normal equations has been described assuming all the data were to be given equal weight. If the weights are not unity (but w) the elements of parts 2 and 4 of the matrix are now replaced by the weights, w, and the setting up of the matrix then proceeds as before.

To take account of weighting when setting up the right hand side of equation (5) each M(I,J) is multiplied by its weight and then summed as before.

The matrix of the coefficients of the normal equations is now inverted using a subroutine from the Harwell Program Library (No. MB01A). This subroutine uses the so called triangular decomposition method of matrix inversion [1].

Both the original matrix and the inverted matrix are stored on disk.

Using the inverted matrix S(I), B(J) and MBAR are computed. Subtracting these values from the original M(I,J) gives the error term $\varepsilon(I,J)$ and hence s^2 the estimate of the variance of the errors can be found. From s^2 and the elements of the inverted matrix the

---

1. HMSO: (1961) "Modern Computing Methods". National Physical Laboratory. HMSO, London

confidence limits on S(I), B(J) and MBAR and on the differences between pairs of B(J) are computed.

The output from the program is:-

(1) Tables showing stations and events used and input data, M(I,J), with weights.

(2) Tables showing the residuals $\varepsilon(I,J)$.

(3) Tables showing best estimates of S(I), B(J) an MBAR their variances and 95% confidence limits.

(4) Tables showing differences between each pair of B(J) and the 95% confidence limits of these differences.

(5) Table showing variables used in the calculations.

Although the program described here has been written to solve a particular problem the program could equally well be used for any problem that can be expressed in terms of the model given in Section 2; this is the familiar two way analysis of variance model.

To carry out a two way analysis of variance using the usual techniques the values of all the elements in the analysis of variance table must be known (one or two missing values can be tolerated). No such restriction applies to this program.

APPENDIX A

MATRICES AND MATRIX INVERSION

A matrix is an array of numbers of the form

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}$$

Unlike determina: s matrices cannot be evaluated to give a single value. They can however be represented by a single quantity, say A, and as such used in many algebraic operations just as if A were a single number.

For example the addition of the matrices A and B means summing corresponding elements of the two matrices. Thus if

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

and

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A + B = \begin{bmatrix} a_{11} + b_{11}, & a_{12} + b_{12} \\ a_{21} + b_{21}, & a_{22} + b_{22} \end{bmatrix}$$

Multiplication of two matrices is more complicated. Thus,

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21}, & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21}, & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Each element of the new matrix is formed by multiplying each element in a row of the first matrix by the corresponding element in the column of the second matrix and summing. For multiplication to be possible the second matrix (matrix B) must have the same number of rows as the first matrix has columns.

Matrices find their widest application in the solution of linear equations.

Consider the equations

$$
\begin{array}{lllllll}
a_{11}x_1 & + a_{12}x_2 & + a_{13}x_3 & \cdot\;\cdot\;\cdot & a_{1n}x_n & = & y_i \\
a_{21}x_1 & + a_{22}x_2 & + a_{23}x_3 & \cdot\;\cdot\;\cdot & a_{2n}x_n & = & y_2 \\
\cdot & \cdot & \cdot & & \cdot & & \cdot \\
\cdot & \cdot & \cdot & & \cdot & & \cdot \\
a_{n1}x_1 & + a_{n2}x_2 & + a_{n3}x_3 & \cdot\;\cdot\;\cdot & a_{nn}x_n & = & y_n.
\end{array}
\qquad \ldots\ldots (A1)
$$

These equations can be represented in matrix for as

$$
\begin{bmatrix}
a_{11} & a_{12} \cdot & \cdot & a_{1j} & \cdot\;\cdot\;\cdot & a_{1n} \\
a_{21} & a_{22} \cdot & \cdot & \cdot & & \cdot \\
\cdot & \cdot & & \cdot & & \cdot \\
\cdot & \cdot & & \cdot & & \cdot \\
a_{i1} & a_{i2} \cdot & \cdot & a_{ij} & \cdot\;\cdot\;\cdot & a_{in} \\
\cdot & \cdot & & \cdot & & \cdot \\
a_{n1} \cdot & \cdot & \cdot & a_{nj} & \cdot\;\cdot\;\cdot & a_{nn}
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_n \end{bmatrix}
=
\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}
\qquad \ldots\ldots (A2)
$$

$$Ax = y.$$

Strictly the term matrix is applied only to A; the column matrices x and y are called vectors.

The solution of (A2) can be represented symbolically as

$$x = A^{-1}y, \qquad \ldots\ldots (A3)$$

where A^{-1} is called the inverse matrix of A.

Now just as in ordinary algebra

$$aa^{-1} = 1,$$

so a unit matrix I can be defined such that

$$AA^{-1} = I, \qquad \ldots\ldots (A4)$$

where

$$I = \begin{bmatrix} 1 & 0 & 0 & . & . & 0 \\ 0 & 1 & 0 & . & . & 0 \\ 0 & . & 1 & & & . \\ . & . & . & & & . \\ 0 & 0 & 0 & . & . & 1 \end{bmatrix}$$

all the elements of I are zero except the diagonal elements which are unity. Operating on a matrix with I leaves the matrix unchanged, i.e., AI = A.

Equation (A4) provides a way of determining A^{-1}. Let c_{ij} be the element of the inverse matrix; then writing out (A4) in full

$$\begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & & a_{2n} \\ . & . & & & & . \\ . & . & & & & . \\ a_{n1} & a_{n2} & . & . & & a_{nn} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & . & . & c_{1n} \\ c_{21} & c_{22} & . & . & c_{2n} \\ . & . & & & . \\ . & . & & & . \\ c_{n1} & c_{n2} & . & . & c_{nn} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & . & . & 0 \\ 0 & 1 & 0 & 0 & . & . & 0 \\ 0 & 0 & 1 & . & . & . & . \\ . & & & & & & . \\ 0 & . & . & . & . & . & 1 \end{bmatrix}$$

Carrying out the multiplication of the left hand side. The result of multiplying the a_{ij} matrix by the first column of the C matrix is

$$\begin{aligned} a_{11}c_{11} + a_{12}c_{21} \quad . \quad . \quad . \quad . \quad a_{1n}c_{n1} &= 1 \\ a_{21}c_{11} + a_{22}c_{21} \quad . \quad . \quad . \quad . \quad a_{2n}c_{n1} &= 0 \\ . \qquad\quad . \qquad\qquad\qquad . \quad & \\ . \qquad\quad . \qquad\qquad\qquad . \quad & \\ a_{n1}c_{11} + a_{n2}c_{21} \quad . \quad . \quad . \quad . \quad a_{nn}c_{n1} &= 0. \end{aligned}$$

This group of n equations has n unknowns $c_{11}, c_{21}, c_{31} \; . \; . \; c_{n1}$: the elements of the first column of the C matrix, $c_{11}, c_{21}, . \; . \; . \; c_{n1}$ can therefore be obtained by solving this set of equations.

A similar group of equations can be obtained by multiplying the A matrix and the second column of the C matrix. The elements in each column of the C matrix can therefore be evaluated in turn; this results in the inverse matrix.

The right hand side of equation (A2) can then be operated on with the inverse matrix to give x as shown in equation (A3).

Writing out (A3) in full

$$\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdot & \cdot & \cdot & c_{1n} \\ c_{21} & c_{22} & \cdot & \cdot & \cdot & c_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ c_{n1} & c_{n2} & & & & c_{nn} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}$$

i.e.,

$$\begin{aligned} x_1 &= c_{11}y_1 + c_{12}y_2 + c_{13}y_3 \cdot \cdot \cdot \cdot \; c_ny_n \\ x_2 &= c_{21}y_1 + c_{22}y_2 + c_{23}y_3 \cdot \cdot \cdot \cdot \; c_{2n}y_n \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{aligned} \qquad \ldots.(A5)$$

As the c's are known and the y's are known the equations (A5) give $x_1, x_2, \ldots, x_n$ the solutions of (A1).

Shorter methods of solving linear equations are available, e.g., Gaussian elimination but matrix inversion has advantages if several sets of equations have to be solved with the same left hand side but different right hand side. Once the inverse of a particular left hand side has been computed it can be used to solve any number of sets of equations simply by operating on the y matrices. Another advantage of the inverse matrix is that it allows the confidence limits of the unknowns to be computed easily in least squares problems (see Section 5).

APPENDIX B

SOME STATISTICAL CONCEPTS AND PROOFS

B1. EXPECTATION

The expectation of a random variable x, usually written E [X], is defined as

$$E[X] = \int_{-\infty}^{\infty} xp(x)dx,$$

where p(x) is the probability that the random variable will take the particular value x. The expectation corresponds to the mean of the whole population of x. Means determined from a set of sample values of x will not usually coincide with E[X] but will approach E[X] as the sample size increases.

The expectation of a constant is the constant, since the constant can take only one value. The expectation of an expected value, E[E[X]], is simply E [X] since E[X] is a constant and as shown above this only has one value.

B2. VARIANCE

Variance measures the spread of a distribution and can be defined on terms of expectation thus,

$$V[X] = E\left[(X - E[X])^2\right], \qquad \text{...... (B1)}$$

or in words the variance is the expected value (average value) of the squared deviation of a random variable from its expectation.

An alternative form of (B1) is

$$V[X] = E[X^2] - (E[X])^2. \qquad \text{...... (B2)}$$

If a and b are constants the variance of a linear function of X, say a + bX, is

$$\begin{aligned} V[a + bX] &= E[((a + bX) - (E[a + bX]))^2] \\ &= E[a^2] + 2abE[X] + b^2E[X^2] - a^2 - 2abE[X] \\ &\qquad - b^2(E[X])^2 \\ &= b^2\{E[X]^2 - (E[X])^2\} \\ &= b^2V[X]. \end{aligned}$$

If Z is the difference of two random variables X and Y, i.e., $Z = X - Y$

$$
\begin{aligned}
V[Z] &= E[Z^2] - (E[Z])^2 \text{ using (B2)} \\
&= E[(X - Y)]^2 - (E[X - Y])^2 \\
&= E[(X^2 - XY + Y^2)] - (E[X])^2 + 2E[X]E[Y] \\
&\qquad - (E[Y])^2 \\
&= E[X]^2 - (E[X])^2 + E[Y^2] - (E[Y])^2 \\
&\qquad - 2(E[X\,Y] - E[X]E[Y]) \\
&= V[X] + V[Y] - 2\,\mathrm{Cov}\,[X,Y].
\end{aligned}
$$

Cov [X,Y] is called the covariance of X and Y and is defined as $E[XY] - E[X]E[Y]$.

The results given above can be generalised for any linear combination of random variables. Thus, if

$$Z = a_o + a_1X_1 + \ldots . a_nX_n$$

$$V(Z) = \sum_i^n a_i^2 V[X_i] + \sum_i^n \sum_j^n a_i a_j \,\mathrm{Cov}\,[X_i, X_j]$$

$$i \neq j.$$

If the random variables are uncorrelated their covariances are zero. If also $V[X_i]$ is constant and equal to σ^2 for all i then

$$V(Z) = \sigma^2 \sum_i^n a_i^2. \qquad \ldots\ldots (B3)$$

APPENDIX C
PROGRAM LISTING

```
B        JOB,                                              LSMF PROGRAM
B        TYPE,COMPILGO,F4
T        SUBTYPE,FIOD
B       9IOD,DISK
         END
T        SUBTYPE,FORTRAN,LMAP,LSTRAP
C
C
C
C
C
C
C
C
C
C
C
C     MAIN LSMF
C     *********
C
C     FINDS DATE AND SETS UP NUMBER ARRAY NUM AND STUDENTS T TABLE FROM CARDS
C
      COMMON              DATE,HEAD(9),NUM(60),ANT(60),N,NR,DISK,
     1                    XMEAN,CLM,VARM,RSQD,RBAR,ISTAN,STAN
      COMMON   /MTRCES/   A(261,261),B(261),  P(60,200)
      COMMON   /ARRAYS/   X(261),Y(60),NRS(200),NRB(60),D(261),
     1                    CLX(261),CLY(60),VARX(261),VARY(60)
      COMMON   /CODES/    STN(200),SNAME,NST,  EVENT(60),ECODE,NBT
      COMMON   /STUDT/    ST(57),T,IDF,NDF
      INTEGER DISK
C
      CALL EDUMP
      CALL SDATE(DATE)
C
      DISK = 9
      DO 410 I=1,60
      NUM(I)=I
  410 CONTINUE
C                                        READ STUDENTS T TABLE
      READ  405, (ST(I),I=1,57)
      PRINT 405, (ST(I),I=1,57)
  405 FORMAT(1X,15F5.2)
      READ  404
      PRINT 404
  404 FORMAT(55H-------------------------------------------------------)

C
      CALL LSMF
      RETURN
      END
T        SUBTYPE,FORTRAN,LMAP,LSTRAP
C     LEAST SQUARES MATRIX FACTORISATION PROGRAM  (LSMF)
C     **********************************************
C
C
C        THIS PROGRAM SEPARATES THE MEASURED MAGNITUDES OF EXPLOSIONS
C     INTO THREE QUANTITIES -- AN EXPLOSION TERM, A STATION TERM AND A
C     MEAN EXPLOSION-STATION TERM. IT IS ASSUMED THAT THE MAGNITUDE,
C     M(I,J), OF THE JTH EXPLOSION AT THE ITH STATION CAN BE REPRESENTED
C     BY THE EQUATION --
C
C                         S(I) + B(J) + MBAR + E(I,J) = M(I,J)    .....(1)
C
C     WHERE S(I) IS THE STATION CORRECTION, B(J) THE EXPLOSION TERM,
C     MBAR THE MEAN EXPLOSION-STATION TERM AND E(I,J) IS AN ERROR.
C
C        S(I), B(J) AND MBAR ARE ESTIMATED BY LEAST SQUARES. TO DO THIS
C     IT IS NECESSARY TO MAKE THE FURTHER ASSUMPTIONS THAT THE SUM S(I)
C     IS ZERO AND THE SUM B(J) IS ZERO, OTHERWISE THE PROBLEM CANNOT
C     BE SOLVED.
C
C        THE PROGRAM ALSO DETERMINES THE CONFIDENCE LIMITS ON S(I), B(J)
C     AND MBAR AND THE CONFIDENCE LIMITS ON THE DIFFERENCES BETWEEN EACH
C     PAIR OF EXPLOSIONS -- ASSUMING THE ERRORS E(I,J) ARE NORMALLY
C     DISTRIBUTED.
C
C        ESSENTIALLY THE PROGRAM CARRIES OUT A TWO WAY ANALYSIS OF VARIANCE.
C                             IF ALL STATIONS RECORD ALL EXPLOSIONS THE
C     PROBLEM TO BE SOLVED IS IDENTICAL TO THAT OF THE ANALYSIS OF
C     VARIANCE AND COULD BE TREATED AS SUCH. THIS PROGRAM HOWEVER CAN
C     ALSO HANDLE THE SITUATION WHERE THE DATA IS INCOMPLETE I. E. SOME
C     STATIONS FAIL TO RECORD ALL EXPLOSIONS -- THIS CANNOT BE DEALT
C     WITH BY THE USUAL ANALYSIS OF VARIANCE TECHNIQUES.
C
C        ALTHOUGH THE PROGRAM HAS BEEN WRITTEN TO SOLVE A PARTICULAR
C     PROBLEM THE METHOD IS GENERAL AND THE PROGRAM CAN BE USED TO SOLVE
C     ANY PROBLEM THAT CAN BE EXPRESSED AS A SERIES OF EQUATIONS OF
C     TYPE (1).
C
C        FOR DETAILS OF ANALYSIS OF VARIANCE TECHNIQUES SEE BROWNLEE K.A
C     STATISTICAL THEORY AND METHODOLOGY IN SCIENCE AND ENGINEERING, AND
```

```
C     KEMPTHORNE O. THE DESIGN AND ANALYSIS OF EXPERIMENTS. BOTH BOOKS
C     ARE PUBLISHED IN THE WILEY SERIES IN STATISTICS.
C
C
C*****************************************************************
C*****************************************************************
C
C  NON-STANDARD LIBRARY SUBPROGRAMS USED
C
C     EDUMP    DUMP ROUTINE
C     SDATE    DATE IN ALPHANUMERIC FORM  DD/MM/YY
C     SECCLK   PICKS UP SECONDS CLOCK READING
C     EXIT     ROUTINE TO BRANCH BACK TO MACHINE CONTROL
C
C*****************************************************************
C
C  VARIABLES USED
C
C        A     IS THE MATRIX
C        B     IS THE SUM OF MEASURED MAGNITUDES
C        C     IS THE DIAGONAL OF THE INVERTED MATRIX
C        X     ARRAY CONTAINS STATION (FOLLOWED BY EVENT) CORRECTIONS
C        Y     ARRAY CONTAINS EVENT CORRECTIONS
C      CLX     CONFIDENCE LIMITS OF X
C      CLY     CONFIDENCE LIMITS OF Y
C     VARX     VARIANCE OF X
C     VARY     VARIANCE OF Y
C
C        NST =  TOTAL NUMBER OF STATIONS
C        NBT =  TOTAL NUMBER OF EVENTS
C        NR  =  TOTAL NUMBER OF READINGS
C        NRS =  NUMBER OF EVENTS PER STATION
C        NRB =  NUMBER OF STATIONS PER EVENT
C        NS  =  STATION COUNT
C        NB  =  EVENT COUNT
C        J   =  COLUMN COUNT
C        I   =  ROW COUNT
C        N   =  SIZE OF MATRIX
C
C     MAXIMUM NUMBER OF STATIONS = 200
C     MAXIMUM NUMBER OF EVENTS   =  60
C
C
C*****************************************************************
C*****************************************************************


C
C  THE DATA DECK SHOULD BE MADE UP AS FOLLOWS  --
C
C     1)  HEADING CARD  WHICH WILL BE REPRINTED AT THE TOP OF EACH PAGE
C             PUNCH THE FIRST 72 COLUMNS ONLY  COLS.73-80 FOR CONTINUATION COUNT
C         FOLLOWED BY    ANY NUMBER OF COMMENT CARDS
C                        TILL A ZERO IS ENCOUNTERED IN THE LAST COLUMN
C
C     2)  STATIONS  IN ORDER REQUIRED IN MATRIX
C             STATION CARDS ARE READY-PUNCHED
C         FOLOWED BY    END STATIONS   CARD
C
C     3)  EVENT CODES  IN ORDER REQUIRED IN MATRIX
C             EVENT CODE  (8 CHARS.)     -  START PUNCHING IN COLUMN  7
C             EVENT DATA AND COMMENTS   -  START PUNCHING IN COLUMN 20
C         FOLLOWED BY    END EVENTS   CARD
C
C     4)  MAGNITUDE DATA
C             STATION CODE    COLUMNS  2-6
C             EVENT CODE      COLUMNS 7-14
C             MAGNITUDE       DECIMAL POINT (.) IN COLUMN 20  (3 DP)
C             WEIGHTING FACTOR IN COLUMN 30
C             IF WEIGHTING FACTOR ZERO ASSUMED TO BE 1
C         FOLLOWED BY    END MAGNITUDES   CARD
C
C     OPTION   IF THIS LAST CARD IS BLANK NO COMPUTATION WILL BE DONE
C              AND ONLY THE INPUT DATA PRINTED
C
C     THE MAGNITUDE DATA WILL BE READ IN FASTER IF ARRANGED IN STATION ORDER
C
C     5)  END OF JOB   CARD FOR NORMAL END OF JOB
C             NORMAL END OF JOB WILL ALSO OCCUR IF NO MORE DATA
C
C     REPEAT  1)  TO  4)  FOR ANY NUMBER OF MATRICES
C
C*****************************************************************
C
C  THE INFORMATION IS PRINTED AS FOLLOWS  --
C
C     TABLE 1    SETUP DATA
C         TABLE 1.1    STATIONS                                  (UP TO  5 PAGES)
C         TABLE 1.2    EVENTS                                    (UP TO  2 PAGES)
C
C     TABLE 2    INPUT MATRIX WITH WEIGHTS
C         PRINTS ALL STATIONS WITH EVENTS IN SETS OF 10          (UP TO 30 PAGES)
```

```
C
C      TABLE 3    MATRIX OF RESIDUALS OF MAGNITUDES
C          PRINTED AS TABLE 2    TRUE RESIDUALS ARE STARRED      (UP TO 30 PAGES)
C
C      TABLE 4    ANSWERS
C          TABLE 4.1     STATION CORRECTIONS WITH 95 PERCENT CONFIDENCE LIMITS
C          PRINTS OUT COMPUTED VALUES, STARS SMALLEST VALUE,
C                     NUMBER IN EACH ROW OF RESIDUALS,
C                     95 PERCENT CONFIDENCE LIMITS, AND VARIANCE
C                                                                (UP TO  5 PAGES)
C          TABLE 4.2     BEST ESTIMATE OF MAGNITUDES WITH 95 PERCENT CONFIDENCE
C          PRINT OUT COMPUTED VALUES,                                     ((LIMITS
C                     NUMBER IN EACH COLUMN OF RESIDUALS,
C                     95 PERCENT CONFIDENCE LIMITS, AND VARIANCE
C                                                                (UP TO  2 PAGES)
C
C      TABLE 5    TRIANGULAR MATRIX OF DIFFERENCES OF MAGNITUDES
C          PRINTS OUT DIFFERENCES WITH 95 PERCENT CONFIDENCE LIMITS
C                                                                (UP TO 42 PAGES)
C
C      TABLE 6    VARIABLES USED DURING THE COMPUTATION               (1 PAGE)
C
C***********************************************************************
C
C
      SUBROUTINE LSMF
      COMMON             DATE,HEAD(9),NUM(60),AWT(60),N,NR,DISK,
     1                   XMEAN,CLM,VARM,RSQD,RBAR,ISTAN,STAN
      COMMON   /MTRCES/  A(261,261),B(261),   P(60,200)
      COMMON   /ARRAYS/  X(261),Y(60),NRS(200),NRB(60),D(261),
     1                   CLX(261),CLY(60),VARX(261),VARY(60)
      COMMON   /CODES/   STN(200),SNAME,NST,   EVENT(60),ECODE,NBT
      COMMON   /STUDT/   ST(57),T,IDF,NDF
      INTEGER DISK
C
      DATA  END(8HEND M   ), BLANK(8H        )
C
C                                     READ HEADING CARD AND ANY COMMENTS
 10   CALL SECCLK(TS)
      CALL HEADER
C                                     ZERO MATRICES
 20   DO 40 J=1,261
      DO 30 I=1,261
      A(I,J)=0.
 30   CONTINUE



      B(J)=0.
 40   CONTINUE
      DO 60 J=1,200
      DO 50 I=1,60
      P(I,J)=1000.
 50   CONTINUE
      NRS(J)=0
 60   CONTINUE
      DO 70 I=1,60
      NRB(I)=0
 70   CONTINUE
C                                     SETUP STATIONS AND EVENTS
      CALL SETUP
      N=NST+NBT
C                                     FORM MATRICES
      NR=0
      NS=1
 100  IND=1
      READ  105, SNAME,ECODE,AMAG,NWT
 105  FORMAT(1X,A5,A8,2X,F13.9,I1)
      NR=NR+1
      IF(NWT) 110,110,120
 110  NWT=1
 120  WT=NWT
      IF(SNAME.EQ.STN(NS)) GO TO 210
      IF(SNAME.EQ.STN(NS+1)) GO TO 200
      IF(SNAME.EQ.END) GO TO 300
 150  DO 160 J=1,NST
      IF(SNAME.EQ.STN(J)) GO TO 180
 160  CONTINUE
      IF(SNAME.EQ.BLANK) GO TO 290
      PRINT 175, SNAME
 175  FORMAT(27H1** UNKNOWN STATION NAME - ,A8//)
      GO TO 280
 180  DO 190 I=1,NBT
      IF(ECODE.EQ.EVENT(I)) GO TO 250
 190  CONTINUE
      GO TO 230
 200  NS=NS+1
 210  DO 220 NB=1,NBT
      IF(ECODE.EQ.EVENT(NB)) GO TO 240
 220  CONTINUE
 230  IF(ECODE.EQ.BLANK) GO TO 290
      PRINT 235, ECODE
 235  FORMAT(25H1** UNKNOWN EVENT CODE - ,A8//)
```

```
      GO TO 280
240   J=NS
      I=NB
250   P(I,J)=AMAG
      NRB(I)=NRB(I)+1
      NRS(J)=NRS(J)+1
      I=I+NST
260   A(I,J)=WT
      B(J)=B(J)+AMAG*WT
      GO TO (270,100),IND
270   IND=2
      NB=I
      I=J
      J=NB
      GO TO 260
280   PRINT 285
285   FORMAT(24H THE INCORRECT CARD IS -)
      PRINT 105, SNAME,ECODE,AMAG,NWT
      RETURN
290   CALL INPUT
      PRINT 295
295   FORMAT(30H1** ONLY INPUT PRINT REQUESTED///12H NO SOLUTION)
      GO TO 10
C
300   NR=NR-1
      LS=NST+1
      DO 320 J=1,NST
      AD=0.
      DO 310 I=LS,N
      AD=AD+A(I,J)
310   CONTINUE
      A(J,J)=AD
      A(J,N+1)=AD
      A(N+1,J)=AD
320   CONTINUE
      DO 340 J=LS,N
      AD=0.
      DO 330 I=1,NST
      AD=AD+A(I,J)
330   CONTINUE
      A(J,J)=AD
      A(J,N+1)=AD
      A(N+1,J)=AD
340   CONTINUE
      AD=0.

      AMAG=0.
      DO 350 J=1,N
      AD=AD+A(J,J)
      AMAG=AMAG+B(J)
350   CONTINUE
      A(N+1,N+1)=AD/2.
      B(N+1)=AMAG/2.
      DO 370 J=1,NST
      DO 360 I=1,NST
      A(I,J)=A(I,J)+1.
360   CONTINUE
370   CONTINUE
      DO 390 J=LS,N
      DO 380 I=LS,N
      A(I,J)=A(I,J)+1.
380   CONTINUE
390   CONTINUE
C                                      PRINT INPUT
      CALL INPUT
C                                      SOLVE MATRIX
      N=N+1
      DO 420 J=1,N
      IF(A(J,J)-1.)420,410,420
410   PRINT 415, J
415   FORMAT(20H1** DIAGONAL ELEMENT,I4,6H  ZERO///12H NO SOLUTION)
      GO TO 10
420   CONTINUE
      WRITE (DISK) ((A(I,J),I=1,N),J=1,N)
      CALL SOLVE(B,X,N,D)
      WRITE (DISK) ((A(I,J),I=1,N),J=1,N)
      REWIND DISK
      READ  (DISK) ((A(I,J),I=1,N),J=1,N)
C                                      SETUP STATION AND EVENT ARRAYS
      XMEAN=X(N)
      DO 470 I=1,NBT
      J=I+NST
      Y(I)=X(J)
470   CONTINUE
      STAN=ABS(X(1))
      ISTAN=1
      DO 490 J=2,NST
      IF(STAN-ABS(X(J)))490,490,480
480   STAN=ABS(X(J))
      ISTAN=J
490   CONTINUE
```

```
C                                           PRINT RESIDUALS
      CALL OUTPUT
      READ  (DISK) ((A(I,J),I=1,N),J=1,N)
      REWIND DISK
C                                           WORK OUT STUDENTS T
      T=0.
      NDF=NR-N+2
      IF(NDF) 675,675,620
 620  IF(NDF-30)630,640,640
 630  IDF=NDF
      GO TO 670
 640  IF(NDF-300)650,660,660
 650  IDF=NDF/10+27
      GO TO 670
 660  IDF=57
 670  T=ST(IDF)
C                                           COMPUTE VARIANCES AND CONFIDENCE LIMITS
 675  RBAR=0.
      RSQD=0.
      DO 690 J=1,NST
      DO 680 I=1,NBT
      AD=P(I,J)
      RBAR=RBAR+AD
      RSQD=RSQD+AD*AD
 680  CONTINUE
 690  CONTINUE
      IF(NDF.GT.0) RSQD=RSQD/FLOAT(NDF)
      DO 710 K=1,N
      VARX(K)=RSQD*D(K)
      CLX(K)=T*SQRT(VARX(K))
 710  CONTINUE
      DO 720 I=1,NBT
      J=I+NST
      VARY(I)=VARX(J)
      CLY(I)=CLX(J)
 720  CONTINUE
      VARM=VARX(N)
      CLM=CLX(N)
      DO 740 I=1,NBT
      J=I+NST
      DO 730 LS=1,I
      K=LS+NST
      P(LS,I)=T*SORT(RSQD*ABS(A(K,K)+A(J,J)-2.*A(K,J)))
 730  CONTINUE
 740  CONTINUE


      IF(NDF.GT.0) RSQD=RSQD*FLOAT(NDF)
C                                           PRINT ANSWERS
      CALL TABLE
      CALL TRIANG
      CALL SECCLK(TF)
      TS=TF-TS
      PRINT 885, HEAD,DATE
 885  FORMAT(55H1TABLE 6    VARIABLES USED DURING COMPUTATION          ,
     1   9A8/113X,6HDATE   ,A8////)
      PRINT 886, NR,N,RSQD,RBAR,T,NDF,XMEAN,CLM,TS
 886  FORMAT(
     1 33H            NUMBER OF READINGS  = , I6        //
     2 33H            NUMBER OF UNKNOWNS  = , I6        ////
     3 33H  SUM OF SQUARES OF RESIDUALS  = , F10.6     //
     4 33H               SUM OF RESIDUALS  = , E10.4     ////
     5 33H                     STUDENTS T  = , F6.2      //
     6 33H NUMBER OF DEGREES OF FREEDOM  = , I6        ////
     7 33H    MEAN STATION-EVENT EFFECT  = , F7.3,5H  +/-,F7.5  ////////
     8 33H   TIME TAKEN TO SOLVE MATRIX  = , F7.3, 8H SECONDS)
C
C                                           LOOP FOR NEW MATRIX
      GO TO 10
      END
T        SUBTYPE,FORTRAN,LMAP,LSTRAP
C     LSMF HEADING PRINT ROUTINE
C     *************************
C
C     THIS ROUTINE READS AND PRINTS HEADING CARDS TILL IT FINDS A ZERO IN COL.80
C     THE CONTENTS OF THE FIRST CARD IS STORED IN ARRAY HEAD N.B. COLS.1-72 ONLY
C
      SUBROUTINE HEADER
      COMMON            DATE,HEAD(9)
      DATA  END1(8H        EN),END2(8HD JOB    )
C
      READ  1, HEAD,IT
 1    FORMAT(9A8,7X,I1)
      IF(HEAD(1).EQ.END1.AND.HEAD(2).EQ.END2) CALL EXIT
      PRINT 2, DATE,HEAD
 2    FORMAT(1H1/113X,6HDATE   ,A8////25X,9A8////)
 10   IF(IT) 20,30,20
 20   READ  3, IT
      PRINT 3
 3    FORMAT(79H-------------------------------------------------------
     1-----------------------,I1)
      GO TO 10
```

```
 30    RETURN
       END
T          SUBTYPE,FORTRAN,LMAP,LSTRAP
C      LSMF SETUP ROUTINE
C      *****************
C
C         THIS ROUTINE SETS UP AND PRINTS STATIONS AND EVENTS
C
       SUBROUTINE SETUP
       COMMON             DATE,HEAD(9)
       COMMON   /CODES/   STN(200),SNAME,NST,   EVENT(60),ECODE,NBT
       DATA  END1(8HEND S    ), END2(8HEND EVEN)
C
C                                                SETUP STATIONS
 20    NST=0
       LINE=40
 21    READ  22, SNAME
 22    FORMAT(1X,A5,74H--------------------------------------------------
      1------------------------)
       IF(SNAME-END1)23,30,23
 23    NST=NST+1
       IF(NST-200)26,24,24
 24    PRINT 25
 25    FORMAT(21H1** TOO MANY STATIONS)
       GO TO 41
 26    IF(LINE-40)29,27,27
 27    LINE=0
       PRINT 28, HEAD,DATE
 28    FORMAT(55H1TABLE 1.1     STATIONS                                  ,
      1   9A8/113X,6HDATE   ,A8/
      2   80H CODE STATION           REGION          LATITUDE      LONGITUDE
      3      CORRECTIONS      /)
 29    PRINT 22, SNAME
       LINE=LINE+1
       STN(NST)=SNAME
       GO TO 21
C                                                SETUP EVENTS
 30    NBT=0
       LINE=40
 31    READ 32, ECODE
 32    FORMAT(6X,A8,66H--------------------------------------------------
      1----------------)
       IF(ECODE-END2)33,40,33
 33    NBT=NBT+1
       IF(NBT-60)36,34,34
 34    PRINT 35
 35    FORMAT(19H1** TOO MANY EVENTS)
       GO TO 41
 36    IF(LINE-40)39,37,37
 37    LINE=0
       PRINT 38, HEAD,DATE
 38    FORMAT(55H1TABLE 1.2     EVENTS                                    ,
      1   9A8/113X,6HDATE   ,A8/
      2   80H       CODE          EVENT DATA AND COMMENTS
      3                        /)
 39    PRINT 32, ECODE
       LINE=LINE+1
       EVENT(NBT)=ECODE
       GO TO 31
C
 40    RETURN
C                                                FINISH FOR ERROR
 41    CALL EXIT
       END
T          SUBTYPE,FORTRAN,LMAP,LSTRAP
C      LSMF INPUT PRINT ROUTINE
C      ************************
C
C         THIS SUBROUTINE PRINTS OUT THE MATRIX OF MAGNITUDES (P)
C             WITH  1)  ALL STATIONS
C                   2)  EVENTS IN SETS OF TEN (10)
C
C             PRINTS THE WEIGHT BY EACH MAGNITUDE READ IN
C             ZERO AND BLANK INDICATE NO VALUE
C
       SUBROUTINE INPUT
       COMMON             DATE,HEAD(9),NUM(60),AWT(60)
       COMMON   /MTRCES/  A(261,261),B(261),   P(60,200)
       COMMON   /CODES/   STN(200),SNAME,NST,   EVENT(60),ECODE,NBT
       DIMENSION FCHAR(10)
C
       DATA  (FCHAR(I),I=1,10) (80H          *1         *2         *3         *4
      1   *5         *6         *7         *8         *9       ),  ANUM(8HEVENT    )
C
       J=0
 420   J=J+10
       NB=J-9
       IF(J-NBT)440,440,430
 430   J=NBT
 440   LINE=40
```

```
      DO 480 NS=1,NST
      IF(LINE-40)460,450,450
  450 LINE=0
      PRINT 454, HEAD,DATE,(ANUM,NUM(I),I=NB,J)
  454 FORMAT(55H1TABLE 2     MATRIX OF MAGNITUDES WITH WEIGHTS
     1   9A8/113X,6HDATE   ,A8/
     2   9H  STATION,4X,A5,I3,9(4X,A5,I3))
      PRINT 455, (EVENT(I),I=NB,J)
  455 FORMAT(13X,A8,9(4X,A8))
  460 DO 470 I=NB,J
      INST=I+NST
      WT=A(INST,NS)
      INST=IFIX(WT+0.5)+1
      AWT(I)=FCHAR(INST)
  470 CONTINUE
      PRINT 475, NS,STN(NS),(P(I,NS),AWT(I),I=NB,J)
  475 FORMAT(1X,I3,2X,A6,F6.3,A3,9(3X,F6.3,A3))
      LINE=LINE+1
  480 CONTINUE
      IF(J.NE.NBT) GO TO 420
      RETURN
      END
T        SUBTYPE,FORTRAN,LMAP,LSTRAP
C     LSMF MATRIX INVERSION ROUTINE
C     *****************************
C
C     THE METHOD USED IS CALLED  TRIANGULAR DECOMPOSITION
C        FROM N.P.L. MODERN COMPUTING METHODS
C
C        BASED ON LIBRARY SUBROUTINE MB01A
C
      SUBROUTINE SOLVE(B,X,M,D)
      COMMON  /MTRCES/  A(261,261)
      COMMON  /ARRAYS/  QQQ(902),C(261),IND(261)
      DIMENSION B(M),X(M),D(M)
C
  100 AMAX=0.0                                                          MB01A003
      DO 2 I=1,M                                                        MB01A004
      IND(I)=I                                                          MB01A005
      IF(ABS (A(I,1))-AMAX)2,2,3                                        MB01A006
    3 AMAX=ABS (A(I,1))                                                 MB01A007
      I4=I                                                              MB01A008
    2 CONTINUE                                                          MB01A009
      MM=M-1                                                            MB01A010
      DO 111 J=1,MM                                                     MB01A011

      IF(I4-J)6,6,4                                                     MB01A012
    4 ISTO=IND(J)                                                       MB01A013
      IND(J)=IND(I4)                                                    MB01A014
      IND(I4)=ISTO                                                      MB01A015
      DO 5 K=1,M                                                        MB01A016
      STO=A(I4,K)                                                       MB01A017
      A(I4,K)=A(J,K)                                                    MB01A018
      A(J,K)=STO                                                        MB01A019
    5 CONTINUE                                                          MB01A020
    6 AMAX=0.0                                                          MB01A021
      J1=J+1                                                            MB01A022
      DO 11 I=J1,M                                                      MB01A023
      A(I,J)=A(I,J)/A(J,J)                                              MB01A024
      DO 10 K=J1,M                                                      MB01A025
      A(I,K)=A(I,K)-A(I,J)*A(J,K)                                       MB01A026
      IF (K-J1)14,14,10                                                 MB01A027
   14 IF(ABS (A(I,K))-AMAX)10,10,17                                     MB01A028
   17 AMAX=ABS (A(I,K))                                                 MB01A029
      I4=I                                                              MB01A030
   10 CONTINUE                                                          MB01A031
   11 CONTINUE                                                          MB01A032
  111 CONTINUE                                                          MB01A033
   65 DO 140 I1=1,MM                                                    MB01A034
      I=M+1-I1                                                          MB01A035
      I2=I-1                                                            MB01A036
      DO 41 J1=1,I2                                                     MB01A037
      J=I2+1-J1                                                         MB01A038
      J2=J+1                                                            MB01A039
      W1=-A(I,J)                                                        MB01A040
      IF(I2-J2)141,43,43                                                MB01A041
   43 DO 42 K=J2,I2                                                     MB01A042
      W1=W1-A(K,J)*C(K)                                                 MB01A043
   42 CONTINUE                                                          MB01A044
  141 C(J)=W1                                                           MB01A045
   41 CONTINUE                                                          MB01A046
      DO 40 K=1,I2                                                      MB01A047
      A(I,K)=C(K)                                                       MB01A048
   40 CONTINUE                                                          MB01A049
  140 CONTINUE                                                          MB01A050
      DO 150 I1=1,M                                                     MB01A051
      I=M+1-I1                                                          MB01A052
      I2=I+1                                                            MB01A053
      W=A(I,I)                                                          MB01A054
      DO 56 J=1,M                                                       MB01A055
      IF (I-J)52,53,54                                                  MB01A056
```

```
   52 W1=0.0
      GO TO 55
   53 W1=1.0
      GO TO 55
   54 W1=A(I,J)
   55 IF(I-1)156,156,57
   57 DO 58 K=I2,M
      W1=W1-A(I,K)*A(K,J)
   58 CONTINUE
  156 C(J)=W1
   56 CONTINUE
      DO 50 J=1,M
      A(I,J)=C(J)/W
   50 CONTINUE
  150 CONTINUE
      DO 60 I=1,M
   63 IF(IND(I)-I)61,60,61
   61 J=IND(I)
      DO 62 K=1,M
      STO=A(K,I)
      A(K,I)=A(K,J)
      A(K,J)=STO
   62 CONTINUE
      ISTO=IND(J)
      IND(J)=J
      IND(I)=ISTO
      GO TO 63
   60 CONTINUE
C
 64   DO 66 J=1,M
      STO=0.
      DO 67 I=1,M
      STO=STO+A(I,J)*B(I)
 67   CONTINUE
      X(J)=STO
      D(J)=A(J,J)
 66   CONTINUE
 68   RETURN
      END
T        SUBTYPE,FORTRAN,LMAP,LSTRAP
C     LSMF OUTPUT PRINT ROUTINE
C     ************************
C
C  THIS ROUTINE COMPUTES AND PRINTS OUT THE MATRIX OF RESIDUALS OF MAGNITUDES  P
C              WITH  1)  ALL STATIONS


C                   2)  EVENTS IN SETS OF TEN (10)
C
C     N.B.  THE TRUE RESIDUALS ARE PRINTED OUT
C      BUT  WEIGHTED RESIDUALS ARE RETURNED TO LSMF
C
      SUBROUTINE OUTPUT
      COMMON               DATE,HEAD(9),NUM(60),AWT(60),N,NR,DISK,
     1                     XMEAN,CLM,VARM,RSQD,RBAR,ISTAN,STAN
      COMMON  /MTRCES/  A(261,261),B(261),  P(60,200)
      COMMON  /ARRAYS/  X(261),Y(60)
      COMMON  /CODES/   STN(200),SNAME,NST,  EVENT(60),ECODE,NBT
      INTEGER DISK
C
      DATA  ANUM(8HEVENT   ), STAR(8H*       ), BLANK(8H        )
C
      J=0
 420  J=J+10
      NB=J-9
      IF(J-NBT)440,440,430
 430  J=NBT
 440  LINE=40
      DO 490 NS=1,NST
      IF(LINE-40)460,450,450
 450  LINE=0
      PRINT 454, HEAD,DATE,(ANUM,NUM(I),I=NB,J)
 454  FORMAT(55H1TABLE 3    MATRIX OF RESIDUALS OF MAGNITUDES
     1   9A8/113X,6HDATE  ,A8/
     2   9H  STATION,4X,A5,I3,9(4X,A5,I3))
      PRINT 455, (EVENT(I),I=NB,J)
 455  FORMAT(13X,A8,9(4X,A8))
 460  DO 470 I=NB,J
      IF(P(I,NS)-1000.)464,462,464
 462  AWT(I)=BLANK
      P(I,NS)=0.
      GO TO 470
 464  AWT(I)=STAR
      P(I,NS)=P(I,NS)-X(NS)-Y(I)-XMEAN
 470  CONTINUE
      PRINT 475, NS,STN(NS),(P(I,NS),AWT(I),I=NB,J)
 475  FORMAT(1X,I3,2X,A6,F8.5,A1,9(3X,F8.5,A1))
      LINE=LINE+1
      DO 480 I=NB,J
      INST=I+NST
      P(I,NS)=P(I,NS)*     A(INST,NS))
 480  CONTINUE
```

```
 490  CONTINUE
      IF(J.NE.NBT) GO TO 420
      RETURN
      END
T        SUBTYPE,FORTRAN,LMAP,LSTRAP
C     LSMF TABLE PRINT SUBROUTINE
C     **************************
C
C        THIS SUBROUTINE PRINTS OUT TABLES OF STATION AND EVENT CORRECTIONS
C
C        THE NUMBER, 95 PERCENT CONFIDENCE LIMITS, AND VARIANCE
C        ARE PRINTED OUT FOR EACH NUMBER
C
      SUBROUTINE TABLE
      COMMON              DATE,HEAD(9),NUM(60),AWT(60),N,NR,DISK,
     1                    XMEAN,CLM,VARM,RSQD,RBAR,ISTAN,STAN
      COMMON   /MTRCES/   A(261,261),B(261),   P(60,200)
      COMMON   /ARRAYS/   X(261),Y(60),NRS(200),NRB(60),D(261),
     1                    CLX(261),CLY(60),VARX(261),VARY(60)
      COMMON   /CODES/    STN(200),SNAME,NST,  EVENT(60),ECODE,NBT
      COMMON   /STUDT/    ST(57),T,IDF,NDF
      INTEGER DISK
C
      DATA  STAR(8H ***     ), BLANK(8H        )
C
C                                         PRINT OUT STATION CORRECTION
      LINE=40
      DO 350 NS=1,NST
      IF(LINE-40)320,310,310
 310  LINE=0
      PRINT 315, HEAD,DATE
 315  FORMAT(55H1TABLE 4.1     STATION CORRECTIONS                     ,
      1   9A8/113X,6HDATE   ,A8/
     2        119H  STATION       COMPUTED      NUMBER        95 PERCENT
     3  VARIANCE                                                        /
     4        119H                     VALUE      IN ROW    CONFIDENCE LIMITS
     5                                                                  )
 320  IF(NS-ISTAN)330,325,330
 325  DIAG=STAR
      GO TO 340
 330  DIAG=BLANK
 340  PRINT 345, NS,STN(NS),X(NS),DIAG,NRS(NS),CLX(NS),VARX(NS)
 345  FORMAT(1X,I3,2X,A5,6X,F6.3,A6,I5,6X,3H+/-,F8.5,6X,F8.6)
      LINE=LINE+1
 350  CONTINUE

C                                         PRINT OUT EVENT CORRECTION
      LINE=40
      DO 380 NB=1,NBT
      IF(LINE-40)370,360,360
 360  LINE=0
      PRINT 365, HEAD,DATE
 365  FORMAT(55H1TABLE 4.2     BEST ESTIMATE OF MAGNITUDES              ,
      1   9A8/113X,6HDATE   ,A8/
     2        119H  EVENT         COMPUTED      NUMBER        95 PERCENT
     3  VARIANCE                                                        /
     4        119H                     VALUE      IN COL.   CONFIDENCE LIMITS
     5                                                                  )
 370  PRINT 375, NB,EVENT(NB),Y(NB),NRB(NB),CLY(NB),VARY(NB)
 375  FORMAT( 1X,I2,2X,A8,4X,F6.3,6X,I5,6X,3H+/-,F8.5,6X,F8.6)
      LINE=LINE+1
 380  CONTINUE
C
      RETURN
      END
T        SUBTYPE,FORTRAN,LMAP,LSTRAP
C     LSMF TRIANGULAR MATRIX PRINT ROUTINE
C     ***********************************
C
C     PRINTS THE LOWER TRIANGULAR MATRIX (P) IN SETS OF 5 EVENTS
C        FOR ALL EVENTS FROM THE START OF THE SET
C
      SUBROUTINE TRIANG
      COMMON              DATE,HEAD(9),NUM(60),AWT(60)
      COMMON   /MTRCES/   A(261,261),B(261),   P(60,200)
      COMMON   /ARRAYS/   X(261),Y(60)
      COMMON   /CODES/    STN(200),SNAME,NST,  EVENT(60),ECODE,NBT
      DATA  ANUM(8HEVENT    )
C
      J=0
 420  J=J+5
      NB=J-4
      IF(J-NBT)440,440,430
 430  J=NBT
 440  LINE=40
      DO 490 NA=NB,NBT
      IF(LINE-40)460,450,450
 450  LINE=0
      PRINT 454, HEAD,DATE,(ANUM,NUM(I),I=NB,J)
 454  FORMAT(55H1TABLE 5       MATRIX OF DIFFERENCES OF MAGNITUDES
      1   9A8/113X,6HDATE   ,A8/
```

```
     2   9H  EVENT   ,8X,A5,I3,4(15X,A5,I3))
         PRINT 455, (EVENT(I),I=NB,J)
   455   FORMAT(17X,A8,4(15X,A8))
   460   DO 470 I=NB,J
         AWT(I)=Y(NA)-Y(I)
   470   CONTINUE
         IF(NA-J)482,482,484
   482   NC=NA
   484   PRINT 485, NA,EVENT(NA),(AWT(I),P(I,NA),I=NB,NC)
   485   FORMAT(1X,I2,2X,A8,5(F11.5,5H  +/-,F7.5))
         LINE=LINE+1
   490   CONTINUE
         IF(J.NE.NBT) GO TO 420
         RETURN
         END
  T         SUBTYPE,DATA
 12.70 4.30 3.18 2.78 2.57 2.45 2.36 2.31 2.26 2.23 2.20 2.18 2.16 2.14 2.13
  2.12 2.11 2.10 2.09 2.09 2.08 2.07 2.07 2.06 2.06 2.06 2.05 2.05 2.05 2.04
  2.02 2.01 2.00 2.00 1.99 1.99 1.99 1.98 1.98 1.98 1.98 1.98 1.98 1.97 1.97
  1.97 1.97 1.97 1.97 1.97 1.97 1.97 1.97 1.96 1.96 1.96 1.96
         START JOB
 RUSSIAN    LOG AMPLITUDE A/T    R1 ONLY    12/01/66          WWSSS
 STU   STUTTGART*          GERMANY          48 46 15.0N     9 16 36.0E
 BOZ   BOZEMAN*            MONTANA          45 36 00.0N   111 38 00.0W
 SCP   STATE COLLEGE*      PENNSYLVANIA     40 48 35.5N    77 52 09.8W
 PRE   PRETORIA*           SOUTH AFRICA     25 45 00.0S    28 15 00.0E
 NUR   NURMIJARVI*         FINLAND          60 30 32.4N    24 39 05.1E
 NAI   NAIROBI*            KENYA             1 16 26.2S    36 48 13.2E
 MAN   MANILA*             PHILIPPINES      14 40 00.0N   121 05 00.0E
 KON   KONGSBERG*          NORWAY           59 38 57.0N     9 37 55.0E
 KEV   KEVO*               FINLAND          69 45 21.2N    27 00 45.1E
 IST   ISTANBUL*           TURKEY           41 02 36.0N    28 59 06.0E
 GOL   GOLDEN*             COLORADO         39 42 01.0N   105 22 16.0W
 GEO   GEORGETOWN*         WASHINGTON DC    38 54 00.0N    77 04 00.0W
 FLO   FLORISSANT*         MISSOURI         38 48 06.0N    90 22 12.0W
 DAL   DALLAS*             TEXAS            32 50 46.0N    96 47 02.0W
 COL   COLLEGE OUTPOST     ALASKA           64 54 00.0N   147 47 36.0W
 BUL   BULAWAYO*           RHODESIA         20 08 36.0S    28 36 48.0E
 ALQ   ALBUQUERQUE*        NEW MEXICO       34 56 30.0N   106 27 30.0W
 SHI   SHIRAZ*             IRAN             29 38 40.2N    52 31 34.1E
 AAM   ANN ARBOR*          MICHIGAN         42 17 59.0N    83 39 22.0W
 MUN   MUNDARING*          AUSTRALIA        31 58 30.0S   116 12 24.0E
 BAG   BAGUIO CITY*        PHILIPPINES      16 24 39.0N   120 34 47.0E
 NOR   NORC*               GREENLAND        81 36 00.0N    16 41 00.0W
 ATL   ATLANTA*            GEORGIA          33 26 00.0N    84 20 15.0W

 MAL   MALAGA*             SPAIN            36 43 39.0N     4 24 40.0W
 TOL   TOLEDO*             SPAIN            39 52 53.0N     4 02 55.0W
 ESK   ESKDALEMUIR*        SCOTLAND         55 19 00.0N     3 12 18.0W
 TRI   TRIESTE*            ITALY            45 42 32.0N    13 45 51.0E
 COP   COPENHAGEN*         DENMARK          55 41 00.0N    12 26 00.0E
 BLA   BLACKSBURG*         VIRGINIA         37 12 40.0N    80 25 14.0W
 ATU   ATHENS UNIV.*       GREECE           37 58 22.0N    23 43 00.0E
 AKU   AKUREYRI*           ICELAND          65 41 12.0N    18 06 24.0W
 KOD   KODAIKANAL*         INDIA            10 14 00.0N    77 28 00.0E
 LON   LONGMIRE*           WASHINGTON       46 45 00.0N   121 48 36.0W
 NDI   NEW DELHI*          INDIA            28 41 00.0N    77 13 00.0E
 POO   POONA*              INDIA            18 32 00.0N    73 51 00.0E
 SEO   SEOUL*              KOREA            37 34 00.0N   126 58 00.0E
 WIN   WINDHOEK*           SOUTH AFRICA     22 34 00.0S    17 06 00.0E
 COR   CORVALLIS*          OREGON           44 35 08.6N   123 18 11.5W
 PEL   PELDEHUE*           CHILE            33 08 37.0S    70 41 07.0W
 ANT   ANTOFAGASTA*        CHILE            23 42 18.0S    70 24 55.0W
 ARE   AREQUIPA*           PERU             16 27 43.5S    71 29 28.6W
 LPB   LA PAZ*             BOLIVIA          16 31 57.6S    68 05 54.1W
 RCD   RAPID CITY*         SOUTH DAKOTA     44 04 30.0N   103 12 30.0W
 TAU   TASMANIA UNIV.*     TASMANIA         42 54 35.7S   147 19 13.5E
 MNN   MINNEAPOLIS*        MINNESOTA        44 54 52.0N    93 11 24.0W
 PMG   PORT MORESBY*       NEW GUINEA        9 24 33.0S   147 09 14.0E
 AQU   AQUILA*             ITALY            42 21 14.0N    13 24 11.0E
 BKS   BYERLY*             CALIFORNIA       37 52 36.0N   122 14 06.0W
 GDH   GODHAVN*            GREENLAND        69 15 00.0N    53 32 00.0W
 CHG   CHIENGMAI*          THAILAND         18 47 24.0N    98 58 37.0E
 CTA   CHARTERS TOWERS*AUSTRALIA            20 05 18.0S   146 15 16.0E
 QUE   QUETTA*             PAKISTAN         30 11 18.0N    66 57 00.0E
 KTG   KAP TOBIN*          GREENLAND        70 25 00.0N    21 59 00.0W
 END STATIONS
       R1150364
       R1160564
       R1190764
       R1161164
       R1040265
       R1030365
       R1110565
       END EVENTS
 STU   R1030365      2.20
 STU   R1150364      2.30
 STU   R1160564      2.20
 STU   R1190764      2.00
 STU   R1161164      2.30
 STU   R1040265      2.21
```

BOZ	R1030365	1.58
BOZ	R1150364	1.70
BOZ	R1160564	1.70
BOZ	R1190764	1.50
BOZ	R1161164	1.76
BOZ	R1110565	0.78
SCP	R1150364	1.30
SCP	R1161164	1.60
PRE	R1030365	1.19
PRE	R1150364	1.08
PRE	R1160564	1.00
PRE	R1190764	0.80
PRE	R1161164	1.11
NUR	R1150364	1.60
NUR	R1160564	1.70
NUR	R1190764	1.50
NAI	R1030365	1.58
NAI	R1150364	1.30
NAI	R1160564	1.40
NAI	R1040265	1.30
MAN	R1030365	2.09
MAN	R1150364	2.20
MAN	R1190764	2.00
MAN	R1161164	2.20
MAN	R1040265	2.20
KON	R1030365	1.67
KON	R1160564	1.90
KON	R1110565	1.00
KEV	R1150364	1.70
KEV	R1160564	1.60
IST	R1030365	1.62
IST	R1150364	1.80
IST	R1160564	1.70
IST	R1190764	1.70
IST	R1161164	1.84
GOL	R1030365	1.18
GOL	R1150364	1.08
GOL	R1160564	1.08
GOL	R1190764	0.90
GOL	R1161164	1.36
GEO	R1150364	1.20
GEO	R1160564	1.30
FLO	R1150364	1.08
FLO	R1160564	1.08
FLO	R1161164	1.23

DAL	R1150364	1.08
DAL	R1161164	1.41
COL	R1030365	1.85
COL	R1150364	1.90
COL	R1160564	2.00
COL	R1190764	1.80
COL	R1161164	1.97
COL	R1110565	1.18
BUL	R1030365	1.30
BUL	R1150364	1.50
BUL	R1160564	1.50
BUL	R1190764	1.40
BUL	R1161164	1.37
BUL	R1040265	1.48
ALQ	R1030365	0.70
ALQ	R1150364	0.90
ALQ	R1160564	0.90
ALQ	R1190764	0.80
SHI	R1030365	1.81
SHI	R1160564	1.90
SHI	R1190764	1.80
SHI	R1161164	1.91
SHI	R1110565	0.81
AAM	R1030365	1.30
AAM	R1160564	1.50
AAM	R1190764	1.56
AAM	R1161164	1.55
MUN	R1160564	1.50
MUN	R1190764	1.40
MUN	R1161164	1.53
BAG	R1150364	1.77
BAG	R1160564	1.80
BAG	R1190764	1.60
NOR	R1190764	1.73
NOR	R1161164	1.73
ATL	R1160564	1.08
ATL	R1161164	1.15
MAL	R1160564	1.80
MAL	R1190764	1.60
MAL	R1161164	1.74
TOL	R1160564	2.09
TOL	R1190764	1.90
TOL	R1161164	2.00
ESK	R1030365	1.83
ESK	R1160564	2.00

ESK	R1161164	1.91
ESK	R1110565	0.90
TRI	R1160564	1.80
TRI	R1190764	1.70
COP	R1190764	1.90
COP	R1040265	2.06
BLA	R1161164	1.18
ATU	R1040265	1.90
ATU	R1161164	1.78
AKU	R1030365	1.90
AKU	R1161164	1.95
KOD	R1030365	1.82
KOD	R1161164	1.94
LON	R1030365	1.53
LON	R1161164	1.65
LON	R1110565	0.81
NDI	R1110565	1.84
NDI	R1030365	2.71
POO	R1030365	1.60
POO	R1161164	1.60
POO	R1040265	1.70
SEO	R1161164	1.34
WIN	R1161164	1.30
COR	R1161164	1.95
PEL	R1161164	1.40
ANT	R1040265	1.52
ANT	R1030365	1.48
ARE	R1030365	0.96
LPB	R1161164	0.60
RCD	R1030365	1.30
TAU	R1161164	1.30
MNN	R1161164	2.11
PMG	R1040265	1.82
PMG	R1030365	1.60
PMG	R1150364	1.68
PMG	R1161164	1.78
AQU	R1161164	1.48
BKS	R1161164	1.55
GDH	R1040265	1.72
CHG	R1030365	1.70
CTA	R1030365	1.23
QUE	R1030365	2.06
KTG	R1030365	1.56

END MAGNITUDES

END JOB